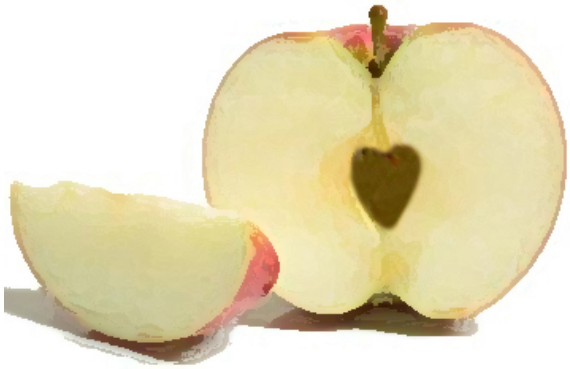


Getting to the Core

Grade 4 Unit of Study

Division

Santa Ana Unified School District Common Core Unit Planner-Mathematics

Unit Title:	**Division**	
Grade Level:	**4th Grade**	**Time Frame: 3 weeks**
Big Idea (Enduring Understandings):	**Quantities can be divided into equal groups.**	
Essential Questions:	• How is an array related to an area model? • How is repeated subtraction related to equal groups in division? What are the patterns that occur? • How can you prove the properties of multiplication and the rules of division? • What is another way to solve a division problem using a different algorithm? • How is the remainder expressed? • Where have you used division to solve a problem in your life?	
21st Century Skills:	**Learning and Innovation:** ☒ Critical Thinking & Problem Solving ☒ Communication & Collaboration ☐ Creativity & Innovation **Information, Media and Technology:** ☐ Online Tools ☒ Software ☐ Hardware	
Essential Academic Language:	**Tier II:** Array Model Multiple Sharing Explain Equal group Remainder	**Tier III:** Inverse operation Dividend Partitioning Properties Divisor Regroup Square number Quotient Divisibility Prime number Factor Composite number Multiplier
What pre-assessment will be given? • Check What You Know quiz, p. 119 (Chapter 6) • Unit 6 prerequisite skills test		**How will pre-assessment guide instruction?** Pre-assessment will determine whether students have the pre-requisite computation skills, academic vocabulary, and level of understanding of new content/standard. If students miss more than two in one skills area provide strategic intervention in a small group.

Instructional Activities

Preparing the Learner Lesson A Launching Mathematical Discourse	Preparing the Learner Lesson B Collaborative Work on Sequencing and Making Equal Shares	Preparing the Learner Lesson C Multiplication Clusters and Mental Math

Preparing the Learner Lessons draw from the Progress to Algebra Continuum for 3rd Grade:
Understand the properties of multiplication and the relationship between multiplication and division.

Arrays: Lessons 1 & 2 **Multiplication and division are different ways to look at the same problem situation.** • Model multiplication and division problems using arrays. • Use arrays to model properties of multiplication in cooperative group.	Grouping: Lessons 3 & 4 **Multiplication and division are different ways to look at the same problem situation.** • Repeated subtraction • Number lines • Making groups • Quick draws	Optional Menu Activities: Lesson 5	Fair Shares: Lessons 6 & 7 **Division is used to solve problems in daily life.** • Quotative- known number in each group • Partitive- known number of groups • Analysis of problem structures.	Remainders: Lessons 8 & 9 **. How the remainder is explained depends upon the problem situation.** • Remainder may be expressed as a fraction or a decimal. • The quotient may be rounded up or down depending upon the problem situation	Optional Menu Activities: Lesson 10
Prime and Composite Numbers: Lessons 11 & 12 **Rules of divisibility are based on number patterns.** • Use arrays to prove numbers are either prime or composite. • Factor table array • Factor trees • Mystery number games ("I'm thinking of"; "I have, who has?")	Multiplication Properties and Divisibility: Lessons 13 & 14 **Multiplication properties demonstrate relationships between numbers.** • Commutative Property • Associative Property • Identity Property • Zero Property • Divisibility Rules • Making Conjectures	Optional Menu Activities: Lesson 15	Dividing Larger Numbers: Optional Lessons 16 – 18 **Division is used to solve problems in daily life.** • Multiples of 10s, 100s, 1000s • Cluster Problems • Inventing Algorithms • Solving Problems with larger numbers • Estimation • Finding an Average	**Unit Assessments** **Summative Multiple Choice** **and** **Performance-Based Task**	

This unit draws from the Progress to Algebra Continuum for 4th grade: Use the four operations (division) with whole numbers to solve problems.

Standards	**Assessment of Standards** (include formative and summative)	
Common Core Learning Standards Taught and Assessed	**What assessment(s) will be utilized for this unit? (F = formative, S = summative)**	**What does the assessment tell us?**
Common Core Mathematics Content Standard(s): **4th Grade Operations and Algebraic Thinking** **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	F: Problem solving journal F: Visual representation of thinking F: Work collaboratively to write the sequence of events in a story, adding a creative ending illustrating how the number of cookies provided and the number of children present affects the number of cookies each child can eat. F: Performance Task, Explaining 13 / 4 = 3 R1 F: Performance Task, How are multiplication and division alike? S: End of Unit Test S: Performance Task, Writing a Division Story for an Expression	Ongoing evidence of students' understanding of the concepts presented Diagnostic information for intervention or acceleration Student comprehension of unit concepts and the big idea: **"Quantities can be divided into equal groups."**
Bundled Language Standard(s): 3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. c. Differentiate between contexts that call for formal English (e.g., presenting ideas) and situations where informal discourse is appropriate (e.g., small-group discussion). 6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases.	F: Teacher evaluation of student use of appropriate mathematical academic language during partner, small group, and class collaborative conversations. F: Use of appropriate academic vocabulary in daily math journals and creation of story problems. S: Use of accurate mathematical terms and appropriate sequential language in culminating written word problem and its solution.	Do students use the appropriate academic language when speaking in class discussions and presentations and when writing in their daily math journals?
Bundled Speaking and Listening Standard(s): 1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 4 topics and texts, building on others' ideas and expressing their own clearly. a. Come to discussions prepared having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles.	<u>Teacher evaluation of student speaking and listening:</u> F: Ask and answer questions in pairs and small groups during and after lessons. F: Work collaboratively to create a tree map of a model for collaborative discussions. F: Participation in presentations of solutions for	When talking about mathematics in pairs and groups, do students follow protocol/rules/ routines for collaborative discussions?

c. Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others. d. Review the key ideas expressed and explain their own ideas and understanding in light of the discussion. 4. Report on a topic or text, tell a story, or recount an experience in an organized manner, using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.	Mystery Numbers. S. Design and write a problem situation to match a given expression, solve the problem, and write a step-by-step explanation of the process used.	Can students plan and deliver an informative presentation with appropriately detailed sequencing? Do all students participate in the thinking, conversation, and final product? Do they follow rules and guidelines for collaboration?

Standards of Mathematical Practice:	(*Check all that apply*) ☒ 1. Make sense of problems and persevere in solving them. ☐ 2. Reason abstractly and quantitatively. ☐ 3. Construct viable arguments and critique the reasoning of others. ☒ 4. Model with mathematics. ☐ 5. Use appropriate tools strategically. ☐ 6. Attend to precision. ☐ 7. Look for and make use of structure. ☒ 8. Look for and express regularity in repeated reasoning.	**Opportunities for Observable Data** *(How will students demonstrate these Mathematical Practices?)* 1. Students will deconstruct the problem. They will make a plan for solving the problem. They will continue with or without support until they reach a solution. 4. Students will solve problems based on a real life situation. 8. Students will notice that repeated subtraction is the same as making equal groups.
Resources & Materials:	**Mathematical Tools:** 1" square tiles, graph paper, index cards, multi-link cubes, base ten blocks, counters, math journals **Media/Technology to be used to deepen learning:** *ST Math Whole Number Multiplication and Division Module*; *Factorization and Prime Numbers*; PowerPoint presentations; NCTM Illuminations Website <http //illuminations.nctm.org> (Multiplication/Division games: *Prime Time*, *Factor Dazzle*, *Times Square*) **Supplementary Materials (lessons from the following resources will be provided):** TERC, *Packages and Groups*; TERC, *Arrays and Shares*; Burns, M., *Math By All Means*, *Division*; Burns, M., *50 Problem-Solving Lessons*; Montgomery, M., *Mystery Numbers*; Equals, *Get It Together*; TERC, *Math Appeal;* Pincezes, E., *One Hundred Hungry Ant;* Pinczes, E., *A Remainder of One;* Hutchins, P., *The Doorbell Rang*	
Interdisciplinary Connections:	**Cite several interdisciplinary or cross-content connections made in this unit of study (i.e. literature, science, social studies, art, etc.)** ELA Theme: money- Student generated word problems using division with money. Science unit: Rocks and Minerals- making arrays for a rock collection.	

Differentiated Instruction:	**Based on desired student outcomes, what instructional variation will be used to address the needs of English Learners by language proficiency level?** • Use of sentence frames (appropriate for language level) to facilitate academic language and conversations. Use of visual organizers to assist processing mathematical ideas • Explicitly teach key academic vocabulary. • Use of manipulatives to facilitate conceptual understanding • Flexible grouping to support language acquisition and target instruction • Use of collaboration to promote socio-cultural learning • Opportunities for verbal rehearsal of concepts	**Based on desired student outcomes, what instructional variation will be used to address the needs of students with special needs, including gifted and talented?** **Special Needs-** • Use of visual organizers in organizing and evaluating evidence. • Explicitly teach key academic vocabulary. • Monitor student responses for corrective teaching • Use of games • ST Math • Opportunities for verbal rehearsal of concepts **GATE-** • Use of pre-assessment results to accelerate/compact curriculum and instruction for students who demonstrate mastery (85% +)

Prerequisite Skills Test

Name _____

Show the expressions using addition:

1. 2 X 4

_____

2. 3 X 5

_____

Solve:

3. How can 7 X 4 be solved using addition?

_____

4. What division does 12 – 3 – 3 – 3 – 3 = 0 show?

_____

5. Mary wants to show 20 ÷ 5 using subtraction. How can she show this?

_____

6. What is the product of 7 X 8?

_____

7. What is the product of 6 and 3?

_____

8. What is 0 X 5?

_____

9. What is 6 ÷ 2?

_____

10. What is 5 ÷ 5?

_____

11. What is 9 ÷ 3?

_____

Name _____

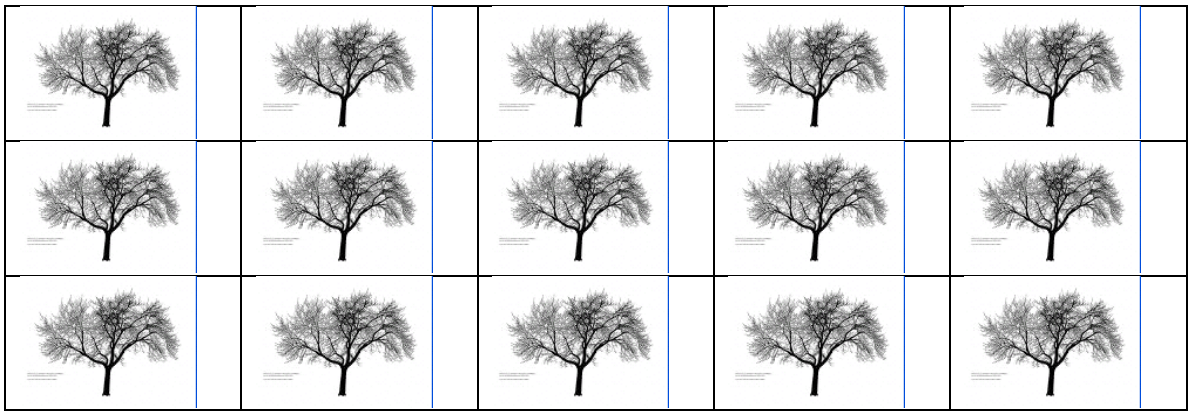
12. Keisha has 9 boxes of crayons with 8 crayons in each box. What number sentence shows how many crayons she has?

_____

13. Each of Eric's four friends gave him 9 marbles. How many marbles did Eric get?

_____

14. Lian decides to plant an orchard. Her orchard looks like this.



What number sentence shows how many trees she has in her orchard?

_____

Find the missing number in each number sentence:

15. 6 X _____ = 24

16. 42 ÷ _____ = 7

Solve:

17. Find the missing number. _____ X 8 = 8 X 5

What property does this number sentence show? _____

Name _____

18. Michelle needs to find the missing number in a number sentence.

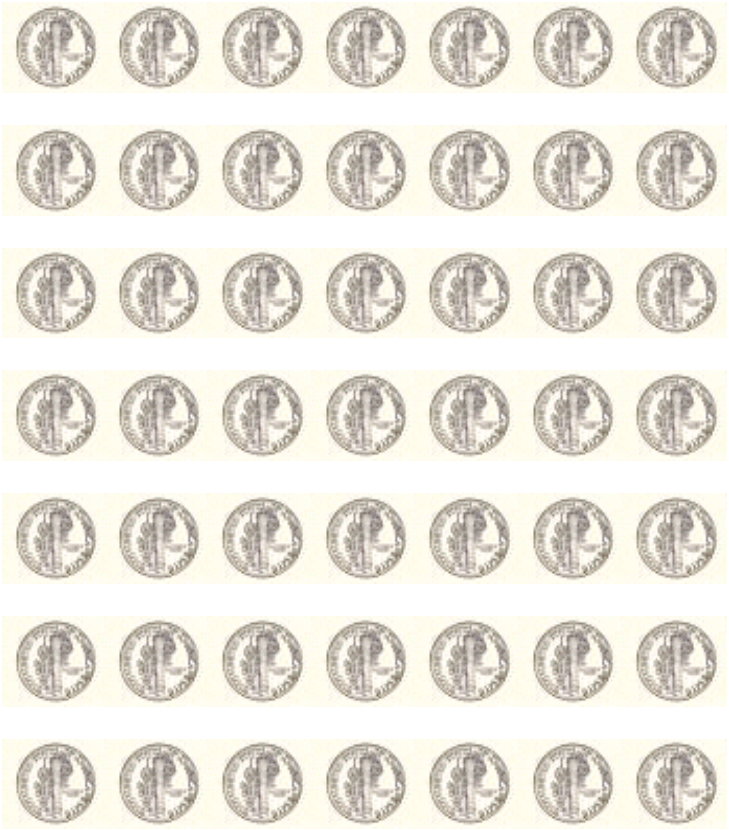
    2 X (5 X 7) = (2 X 5) X _____

    What property does she need to use? _____

19. Find the missing number. 4 X 1 X 8 = 8 X _____ X 1

    What property does the number sentence show? _____

20. Lee had a bag of dimes. She divided the dimes into 7 groups. If every group had 7 dimes in it, how many dimes did Lee have?



Prerequisite Skills Test

Answer Key

1. 2 + 2 + 2 + 2 or 4 + 4
2. 5 + 5 + 5 or 3 + 3 + 3 + 3 + 3
3. 7 + 7 + 7 + 7 or 4 + 4 + 4 + 4 + 4 + 4 + 4
4. 12 ÷ 3
5. 20 – 5 – 5 – 5 – 5
6. 56
7. 18
8. 0
9. 3
10. 1
11. 3
12. 9 X 8 = 72 or 8 X 9 = 72
13. 36
14. 3 X 5 = 15 or 5 X 3 = 15
15. 4
16. 6
17. 5, commutative property
18. 7, associative property
19. 4, commutative property
20. 49

This test measures the following prerequisite skills:

Items 1–3, 6–8, 13, 20—Multiply 1-digit numbers

Items 4–5, 9–11—Divide using repeated subtraction

Items 12, 14–16—Find missing numbers in number sentences

Items 17–19—Recognize and use the commutative and associative properties of multiplication

Any students that miss two or more items in any given area should be given appropriate intervention instruction.

<table>
<tr><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td><td>Unit: Division
Preparing the Learner Lesson # A
Launching Mathematical Discourse</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: Mystery Number cards; Hundred Chart; peanut butter, a knife, and bread
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: Get it Together by Equals;</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:</td><td>Language:</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
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<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
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☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
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<tr><td></td><td>Digits
Multiple
Clue</td><td>Mystery</td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td>Even
Odd
Sum
Difference</td><td>Located
Largest</td></tr>
<tr><td>Pre-teaching Considerations</td><td colspan="3"></td></tr>
</table>

<table>
<tr><th colspan="4">Lesson Delivery</th></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="2">Check method(s) used in the lesson:
☒ Modeling ☐ Guided Practice ☒ Collaboration
☐ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td rowspan="2">Lesson Continuum</td><td>Lesson Opening</td><td colspan="2">Prior Knowledge:
Context and Motivation:
Today we are going to practice having a productive classroom discussion through “math talk”. Math talk is like telling someone else how you made something and why. Who can tell me the steps to making a peanut butter sandwich? (Ask for volunteers. According to responses, act really dense, and do only what they say, with no thought of your own.) “Put the peanut butter on the bread.” (Place the jar of peanut butter on top of the loaf of bread. Children will laugh.) “Open the jar first.” Then what? (Open the jar, then place the open jar on top of the loaf of bread.) “Use the knife,” etc.

If I didn’t know anything about how to make a peanut butter sandwich, could I make one with these directions? Math talk has to be the same way. You have to tell exactly what you did to solve a problem, not leaving out any of the steps.
Today, we’ll practice telling each other what we did to solve a problem.</td></tr>
<tr><td>Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding</td><td>The purpose of this lesson is to launch quality discourse in the mathematics classroom.

<u>Preparation for the lesson:</u> Run enough copies of the Mystery Number cards so that each group will get one set. There are four different sets of Mystery Number cards, so two copies of each set will be sufficient for a class of 32. Cut the six cards apart and place them into an envelope or baggie. Each group will also need one copy of the Hundreds Chart.

<u>Discuss Group Norms:</u>
1) Listen respectfully.
2) Only one person can talk at a time.
3) Everyone must get a turn to speak.

<u>Guided Practice--Fishbowl:</u>
Select one group of three or four students to demonstrate their thinking process, while everyone else watches.
Place this group in the center, with everyone else seated in a circle around them. Give this group a set of Mystery Number cards and a Hundreds Chart.

<u>Instructions:</u>
“In the envelope you will find 6 Mystery Number cards. Each of you will take just one card to begin. Leave the extra cards in the envelope. Each card contains a clue to help you figure out the Mystery Number.”
“Take turns reading your clues out loud, and deciding which numbers fit that clue. Use the Hundreds Chart to help you keep track of all the clues, until you have decided which number fits all the clues.”
“If you get stuck, take another clue out of the envelope.”
“Keep talking about the clues until everyone agrees on the same number.”</td><td>Differentiated Instruction:

English Learners:
Sentence frames using key vocabulary words:
The _____ are 1, 2, 3, 4, 5, 6, 7, 8, and 9. When put together, they form _____. (digits, numbers)

My number is an _____ number. I say it when I count by ___. (even, 2s)

My number is a _____ of ___. I say it when I count by _____. (multiple, 5, etc.)

You find the _____ of two numbers when you _____. (sum, add or difference, subtract)

Special Needs:
Heterogeneous groups to give support for struggling students.

Same sentence frames as given for English Learners.</td></tr>
</table>

		Each member of the group will read their card aloud, and discuss which numbers fit their clue. The teacher will chart the process for the group on chart paper or white board. <u>Fishbowl Reflection:</u> Did the members of this group take turns speaking? Did everyone have a turn to talk? Were the others quiet while one person was speaking? If more than one number fit the clue, how did they decide which number was the correct one? Do you have any suggestions for this group? Who can tell this group one thing they did that made their discussion interesting? <u>Independent Practice:</u> Place students in groups of three or four, with a variety of levels in each group (high, medium, and low, if possible). Make sure that students in each group are seated close enough together to see clearly and to share materials. Pass one Hundreds Chart and one set of Mystery Number cards to each group. Review Instructions: "How many cards are in each envelope? How many cards will each person take to begin? Where will you leave the extra cards? Remember to take turns and make sure everyone has a chance to talk. Keep talking about the clues until everyone agrees on the same number. What will you do if you get stuck?" "When everyone in your group agrees that you have found the right number, write down the steps your group used to solve the mystery." Allow groups to work until they have found the Mystery Number. Then ask groups to record the steps they used to find the Mystery Number. <u>Math Meeting:</u> Bring students together to discuss their solutions and how they worked together. Did the group clearly state each step they followed in finding their Mystery Number?	**Accelerated Learners:** Advanced learners can be asked to create their own clues for a number, and present their clues for others to solve.
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Tim's Number	Tim's Number
Tim's number is a multiple of three. Help your group find Tim's number on the Hundred's Chart.	Tim's number is a multiple of five. Help your group find Tim's number on the Hundred's Chart.
Tim's Number	**Tim's Number**
If you add the digits of Tim's number, you get an odd number. Help your group find Tim's number on the Hundred's Chart.	Tim's number is odd. Help your group find Tim's number on the Hundred's Chart.
Tim's Number	**Tim's Number**
If you multiply the digits of Tim's number together, you get an even number. Help your group find Tim's number on the Hundred's Chart.	Tim's number is near the center of the chart. Help your group find Tim's number on the Hundred's Chart.

Meg's Number	Meg's Number
The sum of the digits of Meg's number is greater than ten. Help your group find Meg's number on the Hundred's Chart.	The difference between the two digits of Meg's number is greater than three. Help your group find Meg's number on the Hundred's Chart.
Meg's Number	**Meg's Number**
Meg's number is a multiple of seven. Help your group find Meg's number on the Hundred's Chart.	The first digit of Meg's number is larger than the second. Help your group find Meg's number on the Hundred's Chart.
Meg's Number	**Meg's Number**
Meg's number is not odd. Help your group find Meg's number on the Hundred's Chart.	Both digits of Meg's number are even. Help your group find Meg's number on the Hundred's Chart.

Paul's Number	Paul's Number
Paul's number is not located on an edge or a corner. Help your group find Paul's number on the Hundred's Chart.	Paul's number is not an even number. Help your group find Paul's number on the Hundred's Chart.
Paul's Number	**Paul's Number**
The difference of the digits in Paul's number is three. Help your group find Paul's number on the Hundred's Chart.	Paul's number is not a multiple of three, five, or seven. Help your group find Paul's number on the Hundred's Chart.
Paul's Number	**Paul's Number**
Paul's number is less than fifty. Help your group find Paul's number on the Hundred's Chart.	The sum of the digits in Paul's number is eleven. Help your group find Paul's number on the Hundred's Chart.

Kelly's Number	Kelly's Number
Kelly's number is a multiple of three. Help your group find Kelly's number on the Hundred's Chart.	The sum of the digits of Kelly's number is even. Help your group find Kelly's number on the Hundred's Chart.
Kelly's Number	**Kelly's Number**
Kelly's number is the largest number on the chart that fits all of the other clues. Help your group find Kelly's number on the Hundred's Chart.	Kelly's number is a multiple of five. Help your group find Kelly's number on the Hundred's Chart.
Kelly's Number	**Kelly's Number**
When you multiply the digits of Kelly's number together, you get an odd number. Help your group find Kelly's number on the Hundred's Chart.	Kelly's number is larger than 50. Help your group find Kelly's number on the Hundred's Chart.

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Collaborative Work on Sequencing and Making Equal Shares</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td></tr>
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Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: The Doorbell Rang, by Pat Hutchins</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will practice working collaboratively and sharing strategies. Students will determine the number of cookies per child based on the number of children and the number of cookies.</td><td>Language:
Students will write the story in sequence and add an effective ending to the story. Students will share their stories orally in group presentations.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
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Division</td><td>WORDS WORTH KNOWING</td></tr>
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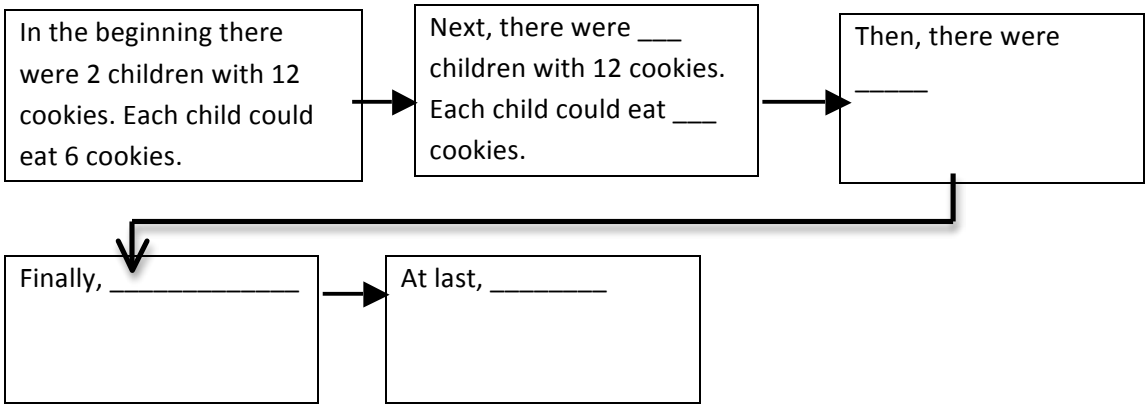
	STUDENTS FIGURE OUT THE MEANING	Equal shares Equal amount The same Divided equally	
Pre-teaching Considerations		This lesson is designed for students who have basic concept of division.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** This lesson is designed for students who have basic concept of division. **Context and Motivation:** "Today we are going to read a story about a family that is sharing cookies with the neighborhood children. How many of you have heard this story before?" (Show the cover of the book.) "Good, you can help us when we are recalling the number of children present and how many cookies each child will get to eat. Now let's read the story." Read the book, *The Doorbell Rang.* "Let's make a flow map to show the sequence of this story, focusing on the number of children present and the number of cookies each child can eat." In the beginning, there were how many children? (2) How many cookies did mother bake? (12) If Mother baked 12 cookies for 2 children, how many cookies could each child eat? (6) Let's put that information into a Flow Map. Start the story with the students, adding transition words, and soliciting from them the number of children and the number of cookies for each child. Ask, "Did the children always share the cookies equally? Did every child get the same amount?"  Gradually release responsibility to the students, drawing information from them. Then ask them to go to their seats and complete the Flow Map, adding information for the number of children and the number of cookies they could eat. "Now we are going to write the end of the story. Work with a partner to rewrite the story, adding a logical ending. You need to tell how many cookies Grandma brought with her, and how many cookies each child could eat. Remember, there were twelve cookies from Mother, plus the additional cookies that Grandma brought. Make sure that the children can share the cookies equally. If there are twelve children, how can we be sure that the children will be able to share all the cookies equally with none left over?" "Oh, the cookies have to be a multiple of twelve, right?"

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	Today's lesson has several purposes: 1) to encourage collaborative work among the students, 2) to write a sequential narration of events using appropriate transitional words 3) to show a relationship between the number of children, the number of cookies, and the number of cookies each child can eat. <u>Guided Inquiry:</u> 1) Form pairs or small groups of three students to work together. Students should be grouped so that everyone can see easily and share materials with one another. 2) Students need paper or math journals for writing. 3) First the students will complete the Flow Map to show the sequence of the story. 4) Then, they will add an ending that tells how many cookies Grandma brought, and how many cookies each child can eat. 5) The final part is to show the relationship between the number of cookies provided, and the number each child can eat. Students may draw a picture, make a table, use tally marks, or show the relationship in a Bridge map Circulate around the room as students are working, asking guiding questions, and encouraging diverse thinking. As you walk around, take note of students who have depicted the information in mathematically powerful ways. Look for organization, clarity, and connections between depictions. The actual work should not take more than about twenty minutes. <u>Math Meeting:</u> Call students together. Ask select students to share their endings to the story. How many cookies did Grandma bring? Students can share their various methods to depict the information. Make connections between different students' work. Ask students to describe how a drawing may show the same information as a table or tally marks. Post sample sentence frames to aid student responses: I like how _____ drew a picture that shows ____ cookies divided among _____ children, just as _____'s table shows the same information. _____ used _____ to show the cookies were divided into _____ equal groups. ___ cookies divided by _____ children was shown by _____ by _____ and by _____.	**Differentiated Instruction:** **English Learners:** _____ children eat _____ cookies. If there were ____ children sharing ____cookies, they would each get ____ cookies. If Grandma brought _____, then there would be _____ and each child could eat _____ . **Special Needs:** Give counters Students may draw their responses. Same sentence frames as given for English Learners.

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	Make a Bridge Map to show the relationship between the number of children sharing cookies and the number of cookies they get to eat. 12 Cookies 2 children — 4 children — 6 children 6 cookies — 3 cookies — 2 cookies Relating factor: "could eat" or "would receive" Have students "read" the Bridge Map. "If there were 2 children sharing 12 cookies, each child could eat 6 cookies. However, if there were 6 children sharing 12 cookies, each child could eat only 2 cookies." Replace numbers of cookies with those given in the student responses. Possible culminating activity: Bring in cookies or brownies for the students to share equally.	**Accelerated Learners:** Choice of number of cookies brought by Grandma. If not a multiple of twelve, how can the cookies be divided equally? Ask students how many cookies would each child get if there were twelve cookies for five children, or for eight or nine? These students can make a Bridge Map to show the relationship between the number of cookies and the number of children.
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

<table>
<tr><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td><td>Unit: Division
Preparing the Learner Lesson # C
Multiplication Clusters</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: Math journals, base 10 blocks may be used, if needed
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: Multiplication Cluster pages</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will solve clusters of multiplication problems that are related in some way.</td><td>Language:
Students will tell and write how they solved the cluster of problems, and how the problems are related.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☒ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td>TEACHER PROVIDES SIMPLE EXPLANATION</td><td>KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td></td><td>Solve
Cluster
Product
Related</td><td></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td></td><td></td></tr>
</table>

<table>
<tr><td colspan="2">Pre-teaching Considerations</td><td colspan="3">This lesson is designed for students who have basic multiplication facts to 10.</td></tr>
<tr><td colspan="5">Lesson Delivery</td></tr>
<tr><td colspan="2">Instructional Methods</td><td colspan="3">Check method(s) used in the lesson:
☒ Modeling ☐ Guided Practice ☒ Collaboration
☐ Independent Practice ☒ Guided Inquiry ☒ Reflection</td></tr>
<tr><td rowspan="2">Lesson Continuum</td><td>Lesson Opening</td><td colspan="3">Prior Knowledge: This lesson is designed for students who have basic multiplication facts to 10.
Context and Motivation:
“How many of you think you can solve tough multiplication problems in your head? Today we are going to solve clusters of multiplication problems in creative ways, without pencil and paper. We will find connections between the problems, and figure out ways to solve problems we never thought we could solve mentally.”

Write 4 X 32 on the whiteboard, then say, “The first problem we will attempt to solve is 4 X 32. How many of you think you can solve that problem in your head without pencil and paper?” Give a few minutes to think.

Then tell the students: A cluster of easier problems that may help us solve this tough problem are: 4 X 3, 4 X 2, 3 X 10, 3 X 40, and 4 X 30. Write these problems on the whiteboard.
Ask students to turn to an elbow partner to discuss the cluster problems and how the problems are related to each other, and how they are related to the original problem. Call on specific students to share their ideas. (4 X 3 is related to 4 X 30, because 30 is ten times as much as 3.)</td></tr>
<tr><td>Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding</td><td colspan="2">The purpose of this lesson is to show how knowing multiplication strategies can lead to competence in division, as well.

Now we will solve other clusters of problems. You and your partner will receive two clusters of multiplication problems. Together, you will decide how the problems are related to each other, then solve each of the problems in the cluster. After you solve each cluster, write about how you solved it. Tell how you used the answer for one problem to help you find the answer to another problem.

Set A: 5 X 3, 5 X 10, 10 X 3, 30 X 5, 50 X 3, 55 X 3

Set B: 2 X 5, 3 X 5, 10 X 5, 30 X 5, 32 X 5

Set C : 5 X 7, 10 X 7, 2 X 7, 20 X 7, 25 X 7

Set D: 3 X 5, 2 X 5, 10 X 5, 20 X 5, 23 X 5

Set E: 3 X 6, 3 X 3, 3 X 10, 6 X 10, 3 X 60, 63 X 3

Set F: 2 X 5, 2 X 4, 2 X 10, 10 X 5, 2 X 50, 2 X 54</td><td>Differentiated Instruction:

English Learners:
Sentence frames:
_____ is like _____ because _____.

I know that _____ groups of _____ is _____ , so _____ groups of _____ would be _____.

If you multiply _____, the product is _____, while if you multiply _____, the product is _____.</td></tr>
</table>

		Set G: 7 X 2, 3 X 2, 7 X 10, 2 X 10, 7 X 20, 2 X 73 Set H: 5 X 3, 6 X 2, 10 X 6, 6 X 5, 60 X 5, 6 X 52 Set I: 8 X 2, 8 X 10, 20 X 8, 10 X 2, 80 X 2, 81 X 2 Set J: 2 X 6, 10 X 6, 3 X 6, 13 X 6, 23 X 6 All of these cluster problems are found on the pages following this lesson. Give one page to each pair of students. Give time for students to solve their cluster, and to write about how they solved the problems. Allow access to base ten blocks or other manipulatives for those students that need them. Post and use these sentence frames to help EL Learners to put their ideas into words. Model how to use the sentence frames with the first cluster you gave students. Tell students they could also use the sentence frames for journal responses. Sentence frames: _____ is like _____ because _____. I know that _____ groups of _____ is _____ , so _____ groups of _____ would be _____. If you multiply _____, the product is _____, while if you multiply _____, the product is _____. <u>Math Meeting:</u> Call students together with their math journals. Ask pairs of students to present their solutions to the cluster of problems they solved. You may ask student presenters to lead the discussion by asking other students to point out connections between the problems. This will involve all students in focusing on the problems presented and their solutions. <u>Guiding Questions:</u> How will solving clusters of problems like this help us to be better problem solvers? Can you think of a time when being able to solve tough multiplication problems in your head might be important? How might these multiplication cluster problems help us with division?	**Special Needs:** Access to base 10 blocks Perhaps just one cluster of problems to solve Work with a partner. Sentence frames as provided for English Learners. **Accelerated Learners:** Students may write their own cluster of problems, and tell how the problems are related.
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Multiplication Clusters

Sets A and B

After you solve each cluster, write about how you solved it. Tell how you used the answer for one problem to help you find the answer to another problem.

Set A
5 X 3 = 5 X 10 = 10 X 3 = 30 X 5 = 50 X 3 = 55 X 3 =
Set B
2 X 5 = 3 X 5 = 10 X 5 = 30 X 5 = 32 X 5 =

Multiplication Clusters

Sets C and D

After you solve each cluster, write about how you solved it. Tell how you used the answer for one problem to help you find the answer to another problem.

Set C
5 X 7 = 10 X 7 = 2 X 7 = 20 X 7 = 25 X 7 =
Set D
3 X 5 = 2 X 5 = 10 X 5 = 20 X 5 = 23 X 5 =

Multiplication Clusters

Sets E and F

After you solve each cluster, write about how you solved it. Tell how you used the answer for one problem to help you find the answer to another problem.

Set E

3 X 6 =
3 X 3 =
3 X 10 =
6 X 10 =
3 X 60 =
63 X 3 =

Set F

2 X 5 =
2 X 4 =
2 X 10 =
10 X 5 =
2 X 50 =
2 X 54 =

Multiplication Clusters

Sets G and H

After you solve each cluster, write about how you solved it. Tell how you used the answer for one problem to help you find the answer to another problem.

Set G

7 X 2 =
3 X 2 =
7 X 10 =
2 X 10 =
7 X 20 =
2 X 73 =

Set H

5 X 3 =
6 X 2 =
10 X 6 =
6 X 5 =
60 X 5 =
6 X 52 =

Multiplication Clusters

Sets I and J

After you solve each cluster, write about how you solved it. Tell how you used the answer for one problem to help you find the answer to another problem.

Set I
8 X 2 = 8 X 10 = 20 X 8 = 10 X 2 = 80 X 2 = 81 X 2 =
Set J
2 X 6 10 X 6 3 X 6 13 X 6 23 X 6

<table>
<tr><td>Unit: Division
Lesson # 1
Arrays</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">Content Standards:
4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: 1” square tiles, graphing paper, counters, realia (egg cartons, six pack of juice)
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: TERC, Math Appeal- "Frog-Gone"(provided in PowerPoint)</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able use arrays to model the relationship between multiplication and division. Students will relate multiplication and division notation representing arrays.</td><td>Language:
Students will be able to explain the relationship between multiplication and division using arrays.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☐ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
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<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td>Academic Vocabulary (Tier II & Tier III)
TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING
Inverse operations
Dimensions
Orientation</td><td>WORDS WORTH KNOWING</td></tr>
</table>

	STUDENTS FIGURE OUT THE MEANING	Array Row Column	Multiplication Division
Pre-teaching Considerations		Knowledge of equal groupings, repeated addition	

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge: Knowledge of equal groupings, repeated addition

Context, Motivation :

Today's big idea is "Multiplication and division are different ways to look at the same problem situation. Both can be represented using _____."

"Today you are going to look for evidence to prove the big idea."

"First let's notice arrays in this poem: (read "Frog-Gone") What about the objects around us?"

(Get student responses, quick draw a few of their responses)

Ask students if they remember what the arrangement is called. Introduce the word dimension into your conversion and write *dimension* to the side of a picture from the story with the description of the array.

Tell students: "Groups of things often come in rectangular arrays. Imagine a six-pack of juice cans. How many rows do you see? How many cans are in each row? Does anyone see it differently?"

On the board begin a chart where you can write the name of the item, write how many total items are in the array, write the dimensions of the array (written in both ways), and draw the array in both orientations. Here's a format you could use:

Item	Total	Dimension	Arrays
Eggs	12	2 rows of 6 2x6 6 rows of 2 6x2	
pack of juice	6		
Box of muffins			

Show the following items as they are discussed: a six-pack of juice cans, eggs, pack of yogurt. (For ELs and visual learners)

Write down some of the students' ideas on your chart. Begin to use the word *dimensions* naturally, just as you have introduced other mathematical terms into the dialogue with your class. Describe the arrays as "3 by 2" or "4 by 3," so students become used to hearing this language.

Show students pictures of simple arrays. Show first 26 slides. Pause for discussion after the orchards. Ask students for ideas to add to our chart.

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

Guided Inquiry

"When a group of people need to sit down, we often arrange chairs in an array that fits the space. Can you think of some of the places this happens?" (movie theater, bus. airplane, school auditorium) Show slides 28-31.

"How could we arrange chairs in a rectangular array for a group of 18 people? How else might 18 chairs be arranged in an array?"

Students, working in pairs, use cubes to make all the arrays they can for 18. Ask each group to choose one of their arrays and cut it out from the graph paper.

Make sure each possible array for 18 is represented. Collect a sample of each array for 18 from the students and post them where all can see them. Some students will see 3 x 6 and 6 x 3 as two different arrays; others will think of them as the same. Allow students to discuss their ideas about this. We suggest that you put up both arrangements, posting pairs next to each other. Arrays provide a model that helps students visualize how multiplication pairs, such as 3 x 6 and 6 x 3, are related. (If you put up all the pairs, you should have 6 arrays: 1 x 18, 18 x 1, 2 x 9, 9 x *2,* 3 x 6, and 6 x 3.)

Ask students to help you label the dimensions of the arrays. This is a good time to establish a class convention for which arrays will be labeled 3 x 6 (3 rows of 6) and which will be labeled 6 x 3 (6 rows of 3). A convention will help students and you communicate with one another more clearly.

Show slides 33 to 42 and assign pairs of students to create arrays for either the doughnut problem or the ornament boxes. (Students could create their arrays with blocks, tiles, in their journals, or on graph paper.

Reflection:

Counting Squares in Arrays Ask students for their ideas about ways to count one of the arrays, for example, the 3 by 6 array. Some students will count the arrays by 1s, counting each individual square. Other students will see that a 3 x 6 array can be counted by 3s (3, 6, 9, 12, 15, 18) or by 6s (6, 12, 18). Sometimes students come up with more inventive ways, such as seeing a 3 x 6 as consisting of two 9s. Emphasize counting by groups by having students look at other arrays for 18. As students make arrays for other numbers in the next activity, continue to encourage them to count their arrays by groups.

"What items have the same number of objects along each side? (For example, 10 x 10) How are these arrays different from other arrays? These are square numbers."

Closing questions:

"How is what we did today related to the big idea: Multiplication and division are different ways of looking at the same problem situation? Both can be represented using _____. What goes in the blank? (arrays) Why? Write about it.
What are some other numbers you would like to investigate?"

Differentiated Instruction:

English Learners:
Use visuals, realia

Special Needs:
Use visuals, realia

Accelerated Learners:
Give students multiple opportunities to explore arrays such as in an online investigation.

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

<table>
<tr><td>Unit: Division
Lesson # 2
More Arrays</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">Content Standards:
4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: 1” square tiles, graphing paper, counters
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module ; http://www.harcourtschool.com/activity/space_arrays/;</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to analyze a number amount to solve a problem designing candy boxes (arrays). Their work focuses on:
• finding factors of numbers
• recognizing prime numbers as numbers with only one pair of factors</td><td>Language:
Students will be able to express their number's factor, product, divisors, dividends, and quotient, and present their findings in a poster.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
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<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
</table>

Academic Vocabulary (Tier II & Tier III)	**TEACHER PROVIDES SIMPLE EXPLANATION**	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
		Factors Prime number Composite number Area model	Inverse Dimension
	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		Notation of dimensions of arrays are determined by their orientation (__ rows of __)	

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**

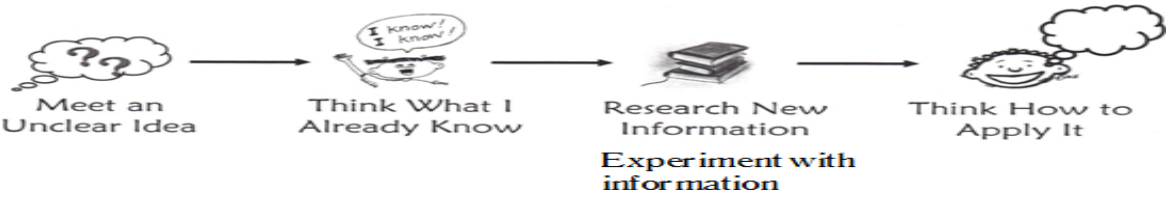
Lesson Continuum

Lesson Opening

Prior Knowledge: Notation of dimensions of arrays are determined by their orientation (__ rows of __)

Context and Motivation:

Today's big idea is "Multiplication and division are different ways to look at the same problem situation."



Meet an Unclear Idea	Think What I Already Know	Research New Information	Think How to Apply It
Ants needed best way to get to the picnic	if they are closer they may move faster∧	Experiment with information 2 by 50 4 by 25 5 by 20 10 by 10∧	Ants travel best in a line

This makes a nice bridge map. The analogy relates to the process the ants went through to learn. They will go through the same process. First present the learning process. Then read the story. Then complete the bridge map.

Today we will investigate the big idea further. We will start by reading "100 Hungry Ants" The power point also shows the story. Stop before you go on to the chocolates.

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**Guided Inquiry** Each student works with a partner to find different ways to form arrays using a given number. At the end of the story are slides of chocolates. Present the context of the activity and then show the slides. Context: ***You are in the candy business. Your company must design candy boxes to package a certain number of chocolates. Each design team must create boxes for a different number of chocolates. Plan all your ideas, draw them, cut them out and paste them on your team's poster.*** **Write the multiplication equations on one side and the division on the other.** **Just as the ants found the arrays for 100, you (students) will find all the arrays for your number of chocolates.** Ask students to contribute any interesting numbers they would like to investigate. We suggest students make arrays for the following size groups: 10, 14 through 30, 32, 36, 40, 42, 44, 45, 48, 49, and 50. Either assign to each pair of students one of these numbers to work on or write the numbers on an index card or post-it and let students choose the number they would like. List on a chart the numbers that students have chosen as a way of keeping track of which numbers have been done. Students can use cubes or tiles to help them plan their arrays. Students should find all the possible arrays for their number and cut out each array from ½ inch graph paper. They glue the arrays onto a larger piece of construction paper to create a "poster" for each number with a title such as "Candy Boxes for 24 Chocolates." They should label each array with its dimensions. Talk with groups as you observe them working: Encourage students to use more than one way of counting, in order to double check the total and to look for ways of checking their arrays by listing them in an organized way, such as 1 x 36, 2 x 18, 3 x 12, 4 x 9, 6 x 6. Students can post finished posters on the wall and choose another number from the list to work on. If all the numbers have been done, suggest that they choose another number (under 50) they would like to investigate. **Gallery Walk:** Place posters around the room accessible to students. Next to each poster place a blank sheet of paper. In pairs, students walk around to view all the array posters and jot down discoveries they have made. Students may add to observations made by others or pose questions. " As you walk around and look at the arrays, here are some things to pay attention to: • **How many arrays does each number have?** (ELD Levels: B EI, I., EA) • **Can you find out anything about even and odd numbers?** (ELD Levels: B. I.) • **See if you can make some discoveries by looking at all our arrays.** (ELD Levels: I, A)	**Differentiated Instruction:** **English Learners:** Using sentence frames Using visuals Using a variety of guided questions: **How can you know when you have found all the arrays for one number?** (ELD Levels: I, EA) **Have you found any numbers that have only one array?** (ELD Levels: I, EA) **How can you count your array to make sure the total is correct?** (ELD Levels: All levels) **Special Needs:** Working in pairs Selecting appropriate numbers Using sentence frames **Accelerated Learners:** Choice of numbers to investigate

		Math Meeting: In a class discussion, students share and discuss these discoveries. Make a graphic organizer of students' observations about the arrays they've constructed. Students might notice: Some numbers have only two arrays (for example, 13 x 1 and 1 x 13). Some students might say these numbers just have one array (since the two are really the same one in a different orientation). Another way to describe these numbers is that they have only two factors, one and the number itself. These special numbers are called *prime numbers.* What prime numbers have students found? Do they know any above 50? Numbers having more than two arrays—or more than two factors—are called *composite numbers.* Since arrays come in pairs (for example, 3 x 4 and 4 x 3), most numbers have an even number of arrays. A few numbers have an odd number of arrays. This occurs when one of the arrays is a square (for example, 10 x 10), which does not have another orientation. Some students may recognize that these numbers, such as 16, 25, and 36, are the *square numbers.*	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

<table>
<tr><td>Unit: Division
Lesson # 3
Multiplication & Division Concept</td><td colspan="2">Grade Level/Course
4th Grade</td><td>Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: 1” square tiles, graphing paper, counters, math journal for note taking
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to solve division problems using a variety of visual strategies.</td><td>Language:
Students will take notes on visual strategies and explain how various strategies are related.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
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☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
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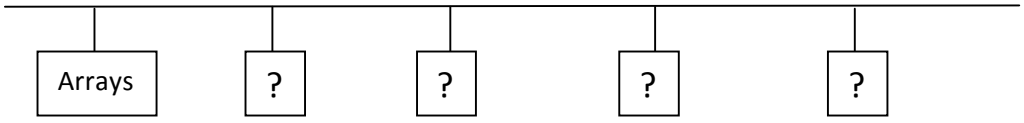
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="2">Academic Vocabulary (Tier II & Tier
TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Number line</td><td>T-Chart</td></tr>
</table>

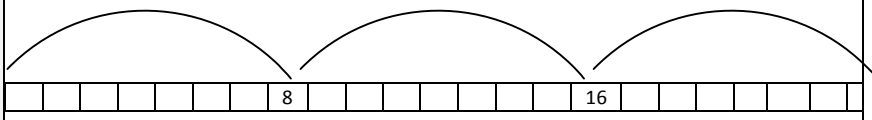
	STUDENTS FIGURE OUT THE MEANING	Equal Groups Repeated Subtraction	Quick Draws
Pre-teaching Considerations		Students should be able to relate multiplication and division as inverse operations.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☒ **Modeling** ☒ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should be able to relate multiplication and division as inverse operations. **Context and Motivation:** Today we want to make new connections to our big idea "Multiplication and division are different ways to look at the same problem situation." (Show students a tree map with the big idea and arrays, and question marks.) **Multiplication and division are different ways to look at the same problem situation.**  Have students create the tree map in their math journals. You will need 8 hangers. What are some things that come in groups? (Anticipate things your students would say come in groups and have some visuals of them accessible: athletic team, six pack of juice, socks, hair ties, pencils, Legos, bag of beads…)	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	Start with a simple division word problem like 48 divided by 8. Put the problem in context using one of the student's ideas i.e., Justin had 48 marbles. He wanted to give some to 8 of his friends so that each friend had the same amount. How many marbles will each friend receive? Ask students how they would interpret the expression 48 ÷ 8. Guide students to create an array of 8 rows of 6. Talk students through the formation of the array: Ask "How many squares should we make in a row? St.: Eight T.: How many more do we need to make? St.: 6 more Guide students to draw the array and label it with students using the correct number sentence 48 ÷ 8 = 6. Ask: How many marbles will each of Justin's friends receive? Sts: They will receive 6 marbles each.	

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

The array model can also be used to represent area. For example, if Justin's friends have 48 square feet in which to play marbles, how much space would each friend be using fairly?

?	1	2	3	4	5	6	7	8
	9	10	11	12	13	14	15	16
	17	18	19	20	21	22	23	24
	25	26	27	28	29	30	31	32
	33	34	35	36	37	38	39	40
	41	42	43	44	45	46	47	**48**
	8							

Have an array ready for the next demonstration. Cut the array by rows and place them together end to end. Mark each group so that it represents a number line.



Have students draw a number line in their journals.

Lead students to create another representation of the problem using a T-Chart (output/input), but complete the chart. Have students add this to their journal.

Number of marbles	groups
8	1
16	2
24	3
32	4
.	.
.	.
.	.

Differentiated Instruction:

English Learners:
Visual representation
Guided teacher modeling of note taking
Use student ideas and verbalizations.
Provide sentence frames.

Special Needs:
Pair student with another student who will be able to provide support during the lesson.
Provide sentence frames.

Accelerated Learners:
Allow these students to work with more challenging numbers.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Model the next three strategies as you did the previous and have students record them in their journals.

Making Groups	Quick Draws	Number Chart
		Students circle every 8th

Now give students a story problem using another "things that come in groups" idea. Do not use 3 digit numbers today. You want to have students experiment with any of the concept building strategies. The following is only an example:

_____(student name) has a bucket of _____(number of objects). He/She wants to share them with ___ (number of people) so that each person receives an equal amount. How many will each friend get?

In pairs, ask students to solve the problem using any two strategies from their journal.
Walk around the room asking students what they are doing and which strategy they are using. If you see students using too much of the same, direct student pairs to try another according to their abilities.

Have a class math meeting:
Choose a few of the students' journals that demonstrate different strategies.
Have students discuss their similarities and differences.

- Making groups is similar to _____ because _____.
- The number line relates to the number chart because _____.
- I thought the _____ strategy relates to the _____ strategy because _____.

Make sentence frames to provide linguistic support for those students who need it.

Lesson Reflection

Teacher Reflection Evidenced by Student Learning/ Outcomes	Go back to the first graphic organizer. Ask students what they should write under the questions marks? Have students share their favorite concept strategy for division: My favorite concept strategy was _____ because _____.

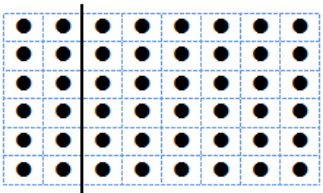
Teacher Sample: Students' final notes should be similar to below. They fold their notes in half and paste one side into journal page so that it will fold out.

Multiplication and division are inverse operations and both can be represented using rectangular arrays or equal groups

Number Line

Counting up or counting down

Array Model

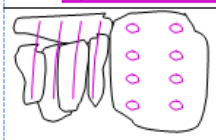


Making Groups:

T-Chart or Input/Output

Number of marbles	groups
8	1
16	2
24	3
32	4
.	.
.	.
.	.

Quick Draws



Area Model: 6 by 8, 6 x 8 = 48; 48 ÷ 8

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	**48**

?

8

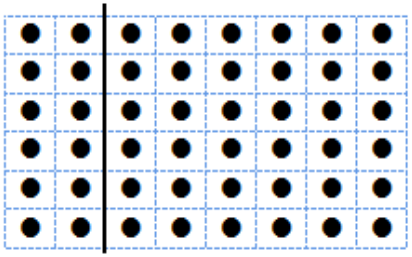
Number Chart

1	2	3	4	5	6	7	**8**	9	10
11	12	13	14	15	**16**	17	18	19	20
21	22	23	**24**	25	26	27	28	29	30
31	**32**	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	**48**	49	50
51	52	53	54	55	56	57	58	59	60

Multiplication and division are inverse operations and both can be represented using rectangular arrays or equal groups

Number Line

Array Model



Area Model

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	**48**

?

8

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

Name: _____

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Lesson # 4
Repeated Subtraction</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
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Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: journal, a box of counters, or cubes
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module;</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to solve division problems using a visual strategies or a number strategy.</td><td>Language:
Students will explain how they used a strategy to solve a division problem with a 3-digit number.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
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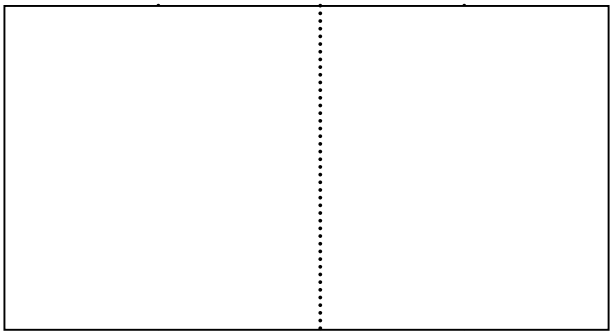
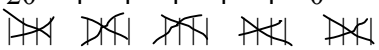
</table>

	STUDENTS FIGURE OUT THE MEANING	Groups of Quick Draws Number line Repeated Subtraction	
Pre-teaching Considerations		Students should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☒ **Modeling** ☒ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Student should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings. **Context and Motivation:** "Multiplication and division are different ways to look at the same problem situation." (Show students a tree with the big idea and the ways to represent the big idea which they completed yesterday.) **Tell students:** Yesterday we made a tree map about how multiplication and division can be represented in groups. We will call them grouping and counting strategies. Today we are going to investigate a number strategy related to division. By the end of today's lesson you should be able to answer the following question. Post the question so students can read it with you. **How is repeated subtraction related to equal groups in division? What are the patterns that occur?**	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**<u>Modeling and Note Taking</u>** Begin with empty comparison map. Have students build a concept foldable in their journals. They should use at least two colors. See example in lesson addendums.  **Remember how <u>MULTIPLICATION is *repeated addition*</u>** *(write on the outside of left side fold)* **Let's draw one group of four...** **Draw another group of four to the picture. And another group. And another group. And yet one more group**. $5 \times 4 = 4 + 4 + 4 + 4 + 4 = 20$. *(write and draw on the inside of left side fold)* || || || || || || || || || ||	**Differentiated Instruction:** **English Learners:** Pair ELD students who may need support with a note taking partner. Using visuals Working in pairs **Special Needs:** Pair Special Needs students who may need support with a note taking partner. Using visuals Working in pairs **Accelerated Learners:** Challenge accelerated learners to represent their thinking in two to three ways of their choice.

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

Let's reverse the process. You start out with 20 sticks. Make one group of four. In your mind, "move it away" from the picture. Form another group of four. Again, "move it away" in your mind, or subtract it from the picture. Keep forming groups of four till you have none left.

$20 - 4 - 4 - 4 - 4 = 0$



(*write and draw on the inside of the right side fold*)

This is repeated subtraction. You subtract 4 repeatedly, or many times, till you hit zero. Each subtraction is forming a group of 4. How many groups did you form? How many times did you subtract? That is the answer to the division problem 20 ÷ 4.

Division is repeated subtraction *(write on the outside of their left side fold)*

$84 \div 21 = ??$

$$\begin{array}{rr} 84 & \\ -\ 21 & 1 \\ \hline 63 & \\ -\ 21 & 1 \\ \hline 42 & \\ -\ 21 & 1 \\ \hline 21 & \\ -\ 21 & 1 \\ \hline 0 & 4 \end{array}$$

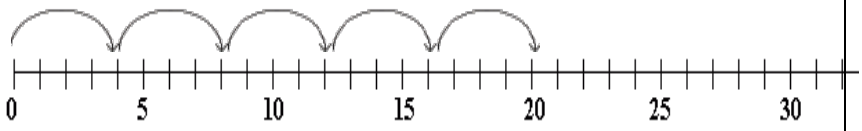
Often, it is handier to actually add instead of subtract Since 13 + 13 = 26, 13 goes to 26 two times. So 26 ÷ 13 = 2
Since 21 + 21 + 21 + 21 = 84, 21 goes to 84 four times.
So 84 ÷ 21 = 4

Write a multiplication sentence AND a division sentence that fits the addition/subtraction facts.

23 + 23 + 23 = __ __ − 23 − 23 − 23 = 0 __ ÷ __ = __ __ ÷ __ = __	40 + 40 = __ __ − 40 − 40 = 0 __ ÷ __ = __ __ ÷ __ = __

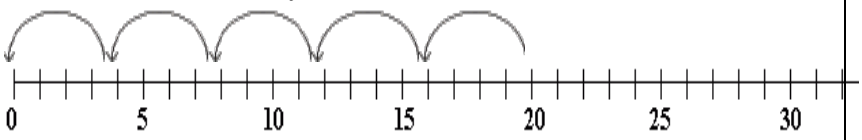
Add one more box to your journal page.

On the left side:
Multiplication is repeated addition, and it is like jumps on the number line.



Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

On the right side:
Division is repeated subtraction. You make jumps of four *backwards* from 20 till you hit 0:



$20 \div 4 = 5$. $20 - 4 - 4 - 4 - 4 = 0$
Five jumps of 4 gets you from 20 to 0.

Guided Inquiry

On the board write the problem $128 \div 8 =$. ___ Ask students how they would interpret this problem. (Think-Pair-Share)
Then show them the box containing 128 cubes.

Tell students:
I have 128 cubes in this box. Some of you suggested that one way of dividing them up would be to share them by dealing out 1 cube to each of 8 people until the box was empty. Sharing is one way of doing division. Another way of dividing up these 128 cubes would be to see how many groups of 8 we can take out of the box. (If someone suggested this method acknowledge it as his or her idea.) Does anyone have any estimates about how many groups of 8 there are in 128?

As students offer their estimates, ask them to explain how they arrived at their number. Record their estimates on the board.

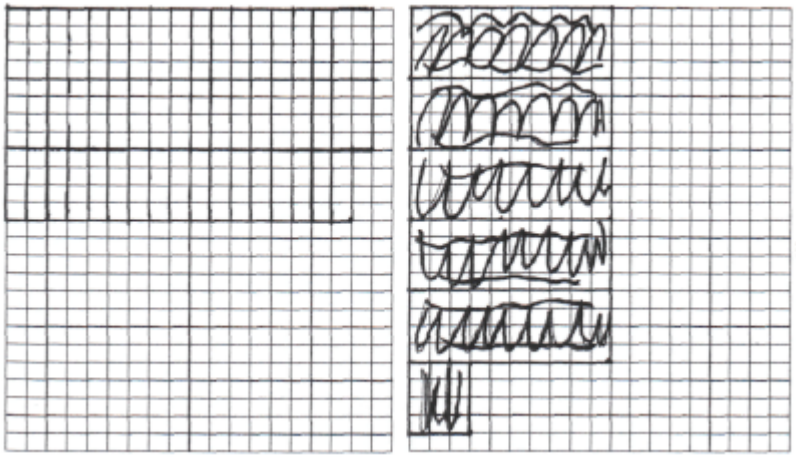
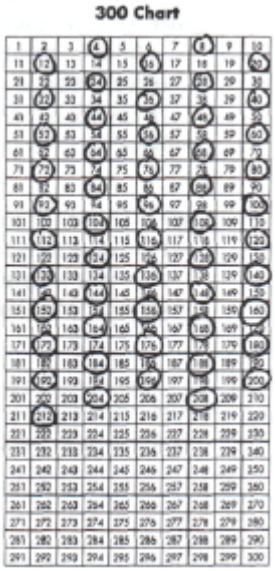
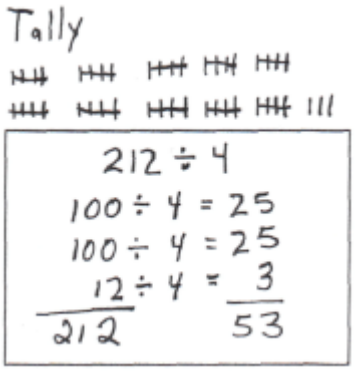
Making Groups of 8:
With the help of a student, begin to remove groups of 8 cubes from the box, putting them in cups or plates and lining them up so students can see them. When you have removed 10 groups of 8, pause and ask the students to figure out how many cubes have been removed from the box.

Let's see, so far we have removed 10 cups of 8 cubes from the box. Talk with the person next to you about how many cubes we have taken out and how many cubes are left in the box.

Guiding Questions:
How many cubes have been removed from the box?
How many cubes are left in the box?
How did you figure this out?

I want to keep track of how many cubes we have removed. Many of you said that you knew 10 groups of 8 was 80, so I'm going to write 10 x 8 = 80.
Write $10 \times 8 = 80$ under the problem $128 \div 8 =$.
Many of you also said there were 48 cubes left in the box. Let's see how many more groups of 8 we can get from 48.

At this point, many students might know there are 6 groups of 8 in 48. Acknowledge their thinking and suggest that you continue to pull out groups of 8 as a way of double-checking. In this way you continue the problem for those students who may not be sure that $6 \times 8 = 48$.

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	When the 48 cubes have been removed, ask a student how many groups of 8 were removed and record this on the board: 128 + 8 = 10 x 8 =80 6 x 8 =48 Count the total number of groups of 8 removed from the box of 128 and record the answer to the problem 128 ÷ 8 = 16. Present a new problem to your students for them to solve with a partner. Record the new problem on the board or overhead: 212 ÷ 4 = **Suppose you had 212 cubes in a box and wanted to see how many groups of 4 cubes you could take out of the box. (If you assembled 212 cubes in a different container, show them to the students.) Work with a partner and figure out how many groups of 4 that would be. When we did the last problem together, I had a way of keeping track of what we were doing. You and your partner should also have some way of keeping track of this problem. Make available any tools students might need to help them solve this problem (cubes, 300 charts, graph paper, paper and pencil).** ***Included is a 300 Chart; some dyads may want to experiment with it.*** **Remind students to double-check their work. Some ways students may solve 212 ÷ 4 = are shown here.**    	

		As students are working on this problem, circulate around to different groups and observe the following: • Are students using cubes? If so how are they using them? Are they making individual groups of 4, or are they pulling out larger groups, such as 40? • How are students keeping track of their work? • Are students breaking apart the problem into more familiar problems such as 200 ÷ 4 and 12 ÷ 4? • Are they using their knowledge about number relationships—for example, that there are twenty-five 4's in 100 or ten 4's in 40? • Are they double-checking their work? Students will vary in their approaches to this problem. While we do want students to use important landmarks and break apart problems, it is most important that students be able to understand what the problem means. Most likely there will be students in your class who will be counting out groups of 4 as a way of solving the problem. Encourage them to start using larger clumps: How many 4's are in 20? In 40? In 100? However, make sure students can really explain what they are doing meaningfully rather than simply copy a strategy you or other students use. **<u>Math Meeting</u>** As pairs of students finish, they can share their strategy with another pair. When most students have completed the task, bring the group members together to share their strategies. Ask pairs of students who used different approaches to share their work and to show how they kept track of their work.	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes		**How is repeated subtraction related to equal groups in division?** **What are the patterns that occur?** **What do you understand better?** **What needs to be clarified for you?**	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300

Division
is
repeated
subtraction

Strategies

212 ÷ 4

↓

100 ÷ 4 = 25

100 ÷ 4 = 25

+ 12 ÷ 4 = + 3

212 53

Mutiplication
is
repeated addition

Number

Journal

Don't forget the number line!

0 5 10 15 20 25 30

5 × 4 = 20

$5 \times 4 = 4+4+4+4+4 = 20$

$4 \times \square = 20$

missing factor

①

$20-4-4-4-4-4=0$

$20 \div 4 = 5$

② Subtracting larger numbers

$84 \div 21 = ??$

84 − 21 = 63	1
63 − 21 = 42	1
42 − 21 = 21	1
21 − 21 = 0	1
	+ 4

Adding can be handy!

13 + 13 = 26, 13 is half of 26 so, 26 ÷ 13 = 2

or

84 ÷ 21 = 4, start with 21 + 21 + 21 + 21

Don't forget the number line!

0 5 10 15 20 25 30

<table>
<tr><td>Unit: Division
Lesson # 5
Menu Activities</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays,
and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Textbook: 4th Grade Houghton Mifflin Intervention Activities
Mathematical Tools: array cards
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: Menu Activities</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to solve division problems using a variety of strategies.</td><td>Language:
Students will explain their thinking while playing division games.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="2">Academic Vocabulary (Tier II & Tier III)
TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2"></td><td></td></tr>
</table>

	STUDENTS FIGURE OUT THE MEANING		

Pre-teaching Considerations	Students should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings.

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☐ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge: Student should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings.
Context and Motivation:
To the Teacher: The following activities are provided so that students have independent opportunities to practice multiplication and division, while the teacher has opportunity to meet with students who are struggling.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Students work on the Menu Activities, while the teacher works with small groups of students.

The following are **Menu Activities**:
Multiplication Pairs
Count and Compare
Small Array/Big Array
100 Hungry Ants: Students work individually or with a partner to explore how to group other numbers of ants. Reread the story or have a class discussion in which the students retell what happened. Have students figure out how many ants were in each line whenever the ants regrouped.
Show the students how rectangular arrays connect to division by using division symbolism to record how to represent each way the ants regrouped.

In students math journals students should keep a record of Menu Activities that they complete.

Intervention Activities from Houghton Mifflin
***Strategic Intervention* Book**:
Groupings pp. 37- 38, 43-44 (for students who are struggling with making equal groups)
Inverse Operations pp. 45 (for students who are struggling with making a connection between multiplication and division)

Differentiated Instruction:

English Learners:

Interventions given in body of lesson.

Special Needs:
Interventions given in body of lesson.

Accelerated Learners:
Activity choices given in body of lesson.

		***Benchmark Intervention* Book**: Model Division p. 56 (for students who are struggling with showing their work using base ten blocks) ***English Learners Resources* Book**: Using Language and Solving Word Problems pp. 80-82 (for Beginning and Early Intermediate English Language Learners) **Can be either Teacher Directed or students can listen to the CD and complete the activities in small groups or pairs.** Call students together to debrief the Menu Activities Session. Ask students: What challenges did you find while working? Is there a learning pair who wants to share how well they worked together? Why? Which activities did you complete? How did the activity help you?	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Starting Menu Activities

If you set up your choices at stations, list the materials students will find at each station. Students can keep track of their choices on their own choice lists.

Choice 1: Array Game: Multiplication Pairs: directions; Array Cards, Sets A and B (1 set per pair)
Choice 2: Array Game: Count and Compare: directions; Array Cards, Sets A and B (1 per pair)
Choice 3: Array Game: Small Array/Big Array: directions; Array Cards, Sets A and B (I set per pair)
Choice 4: Hungry Ants: directions; paper to record (1 per pair)

Make copies of game directions available or simply post each sheet. Students may refer to the directions when in doubt about the rules of the game. Students may choose to play using only the Array Cards in Set A, which consists of multiplication pairs with products up to 50. Then when they feel comfortable, students may include Array Cards from Set B.

Choice 1: Array Game: Multiplication Pairs
Given the dimensions of an array, students are to find the total number of squares in the array; given the total, students are to find the dimensions. As they play, students write the multiplication pairs and relationships they know and don't know on a sheet of paper.

Choice 2: Array Game: Count and Compare
Students use multiplication relationships to find the sizes of students' array cards and then determine the largest.

Choice 3: Array Game: Small Array/Big Array
Students use their array cards to make "matches" between a large array and two or more smaller arrays. Each student should write their "matches" on a sheet of paper using mathematical statements.

Choice 4: Hungry Ants:
Students explore how to group other numbers of ants.

During Choice Time, circulate among the groups and observe students as they are involved with an activity, or use the time to meet with small groups of students who are having difficulty with a particular activity.
Some things you might look for are the following:

- How are students making decisions about choosing an activity and organizing their time and materials?
- Are there too many or not enough activities going on at once?
- Are students keeping track of the choices they have completed?
- How are students figuring out the total number of squares in arrays? Are they counting one by one? Counting by groups? Do they know the multiplication pairs?
- Do some students need to spend more time counting by 2s, 3s, and 6s?

Are some students ready to add the next set of arrays (Set B) to their existing set?

How to Play Multiplication Pairs

Materials

- Set of array cards
- Paper and pencil

Players: 1, 2, or 3

How to Play

1. Spread out all the array cards in front of you. Some should be turned up, showing the dimensions. Others should be turned over to show the total.
2. Choose an array card and put your finger on it. (Don't pick it up until you say the answer.) If the dimensions are showing, you must give the total. If the total is showing, you must say the dimensions of the grid. The shape of the array will help you!

   For example: Suppose you pick an array with total 36 showing. The dimensions could be 6×6, or 9×4, or 12×3. You must decide which is right. The shape of the array is a good clue.
3. Turn the card over to check your answer. If your answer is correct, then pick up the card.
4. If you are playing with a partner, take turns choosing and identifying cards. Play until you have picked up all the cards.

   While you are playing, make lists for yourself of "pairs that I know" and "pairs that I don't know yet." Keep these lists in your math folder.

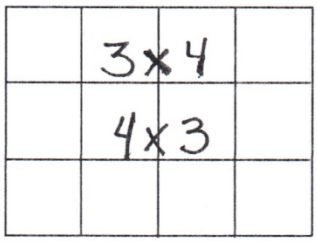
How to Play Count and Compare

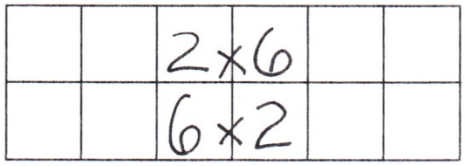
Materials: Set of array cards

Players: 2 or 3

How to Play

1. If you are playing with a partner, sit across from each other. If three people are playing, sit in a circle.
2. Deal out the array cards with the total sides face down. Players should all have the same number of cards. Set aside any that are left over.
3. Place your cards in a stack in front of you, with the total side face down.
4. Players take the top card from their stacks and place these cards side by side (total sides still face down).
5. Decide which array is largest.You can do this just by looking, or by skip counting by rows to find the total of each. Counting the squares by 1's is not allowed.
6. The player with the largest array takes the cards, after proving that it is the largest.
7. Sometimes arrays of the same size may be played in one turn—like this:





When this happens, the players decide together who will get the cards. Once a rule is decided, it cannot be changed until the game is over.

8. The game is over when time is up or one player runs out of cards.

How to Play Small Array/Big Array

Materials: Set of array cards

Players: 2

How to Play

1. Deal out 10 array cards to each player with the dimensions side up. Spread out the cards in front of you.
2. Spread out 6 more cards, dimensions side up, in the center of the table. Place the remaining cards in a deck in the center of the table.
3. The goal is to make a "match" by covering a big array with two or three smaller arrays. Players take turns.
4. From your smaller arrays, choose one that matches one dimension of a big array in the center of the table.
5. If none of your array cards matches, you can choose a card that matches a dimension of the big array from the center of the table. Or you can pick the top card from the deck and play it if you can.
6. If you use an array from the center of the table to cover another array, you can either replace it with a card from the deck, or discard one of your array cards. There should always be 6 cards in the middle.
7. When you cover a big array, you can collect the "match." While playing, keep a list of the dimensions of the large array and the smaller arrays:

$$\begin{aligned} 7 \times 6 &= 3 \times 6 + 4 \times 6 \\ 42 &= 18 + 24 \end{aligned}$$

8. If you run out of cards, take 4 cards from the deck. The game is over when there are no more cards or no more matches can be made.

Hungry Ants

I

1. Figure out what would happen if 20 ants tried to group themselves into 1 line, 2 lines, 3 lines, and so on up to 10 lines. How many ants would be in each line?

   Record your answers like this:

   20 Hungry Ants

   1 line of _____
   2 lines of _____
   3 lines of _____
   4 lines of _____
   5 lines of _____
   6 lines of _____
   7 lines of _____
   8 lines of _____
   9 lines of _____
   10 lines of _____

2. Choose another number of ants and do the activity again.

Unit: Division **Lesson # 6** Equal Sharing Equal Groups	**Grade Level/Course** 4th Grade	**Duration:** 60 min. **Date:**
Common Core Standards	**4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** counters, math journals, index cards **Media/Technology to be used to deepen learning:** ST Math Whole Number Multiplication and Division Module	
Objectives	**Content:** Students will be able to demonstrate partitive (known number of groups) and quotative (known number in each group) division using cubes or tiles. Students will create models of each problem type.	**Language:** Students compare the processes of equal sharing and equal groups in division and create examples of each problem type.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒**Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary TEACHER PROVIDES SIMPLE EXPLANATION	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**

	STUDENTS FIGURE OUT THE MEANING	Equal sharing Equal groups	
Pre-teaching Considerations		Students should have concept of equal groups, meaning of conventional notation (4 x 6 to designate 4 groups of 6)	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☐ **Modeling** ☒ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should have concept of equal groups, meaning of conventional notation (4 x 6 to designate 4 groups of 6) **Context, and Motivation:** Today's big idea is "Multiplication and division are different ways to look at the same problem situation.' Today we will investigate the big idea further. We will learn about different ways to form equal groups. Let's consider these two problems: "I have 18 balloons for my party. After the party is over, I am going to divide them evenly between my sister and me. How many balloons will each of us get?" "I have 18 balloons for my party. After the party is over, I am going to tie them together in bunches of two to give to my friends. How many bunches can I make?" Using counters, represent each of these problem situations. How do they "look" different? Each of these problems is a division situation—a quantity is broken up into equal groups. Although we begin with the same number of balloons, and the notation for each problem is the same (18 ÷ 2 = 9) the problem situation is really quite different. In the first situation, you know the number of groups—2. Your question is "How many balloons will be in each group?" In the second situation, you know that you want 2 balloons in each group, and your question is, "How many groups will there be?" In each case you divide the balloons into equal groups, but the results of your actions look different. In the first case, the balloons are passed out, one at a time to the two children, until they are all gone. In the second case, the balloons are given in pairs to as many children as possible. We could say, " Divide 18 into two groups. How many are in each group?" or "How many 2s are in 18?" We will call the first problem type equal sharing, and the second problem type equal groups.	

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**Guided Practice** Now study these problems. Make a visual representation of each problem in your math journal, and determine if the problem situation is asking for equal sharing (known number of groups) or equal groups (known number in each group). Students are given choices of numbers to use for each problem, depending on their abilities. 1. Peter has 96 (54, 36) photos to place in his photo album. He can fit six photos on each page. How many pages will he need for all of his photos? 2. Galen uses 120 (56, 32) ounces of toothpaste in 8 months. How many ounces of toothpaste did he use each month? 3. Mara's dessert contained 425 (300, 125) calories. She ate five items, and each contained the same number of calories. How many calories are in each item? 4. The elementary school just received 76 (63, 42) reams of paper to divide evenly among the teachers. Each teacher will receive 3 reams of paper. How many teachers work at the school? Monitor students as they work. Make note of student work that is representative of many students' thinking, and work that can push the thinking of the group forward. Strategically sequence student sharing to maximize the impact. A general rule is to begin with students who have the most concrete model, and proceed to those who use a more abstract representation of the problem. Call students together to debrief answers and depictions of problem situations. Focus on bringing out the concepts of equal sharing (known number of groups) and equal groups (known number in each group). Now ask students to work in pairs to create one word problem of each type. Write the problems on index cards. Gather the index cards and pass them out to different students. Students can act out each others' problems.	**Differentiated Instruction:** **English Learners:** Using sentence frames Using visuals Working in pairs **Special Needs:** Working in pairs Modifying numbers given Using sentence frames **Accelerated Learners:** Creativity in creating story problems of own interests and difficulty level
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes		How do we know whether a division problem is asking for equal shares or equal groups? Why does it matter?	

<table>
<tr><td colspan="2">Unit: Division
Lesson # 7
Fair Shares</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
<tr><td colspan="2">Common Core Standards</td><td colspan="3">4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td colspan="2">Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: cubes, counters, math journals, graph paper, calculators, box of 12 pencils
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module</td></tr>
<tr><td colspan="2">Objectives</td><td colspan="2">Content:
Students will be able to apply partitioning (known number of groups) and sharing (known number in each group) division to real life problem situations.</td><td>Language:
Students describe situations in which equal sharing or equal groupings are more appropriate.</td></tr>
<tr><td colspan="2">Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td colspan="2">Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
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<tr><td colspan="2">Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td>TEACHER PROVIDES SIMPLE EXPLANATION</td><td>KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td colspan="2">WORDS WORTH KNOWING</td></tr>
<tr><td></td><td>Equation</td><td colspan="2"></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td>Dozen</td><td colspan="2">Package</td></tr>
</table>

Pre-teaching Considerations	Students should have the concept of packages containing equal amounts. 12 is the same as one dozen.	
Lesson Delivery		
Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum — **Lesson Opening**	**Prior Knowledge:** Students should have the concept of packages containing equal amounts. 12 is the same as one dozen. **Context and Motivation:** Today's big idea is "Multiplication and division are different ways to look at the same problem situation." Today we will investigate the big idea by exploring a real life problem for our class. It is time to order pencils, and we will need to order enough so that everyone in the class has six pencils. How can we decide how many pencils that might be? Show a box of pencils to the class. "Pencils come in packages of 12. What is another way to say 12? That's right, one dozen." 1. How many packages would we have to order to give 2 pencils to each student in our class? 2. How many packages would we have to order to give 4 pencils to each student in our class? 3. How many packages would we have to order to give 6 pencils to each student in our class?	
Lesson Continuum	**<u>Guided Inquiry</u>** Students work in pairs, in small groups, or independently to complete this investigation. Tell students they must write an equation for each problem and show how they solved it. They can use words or pictures as a way of illustrating their work. Some students may wish to use cubes, graph paper, or calculators. These mathematical tools should be available to them. Remind all students to record their solutions using numbers in some way. Students may choose to use multiplication or division to show their work. Addition may also be integrated into the work of some students. **<u>Guiding Questions:</u>** • In deciding how many pencils to order, are we thinking about passing out the pencils to each student one-by-one, or are we thinking about giving a certain number of pencils to each student? • How many pencils would we need if each student only received one pencil? How many packages of 12 is that? • How do we keep track of the number of pencils needed and the number of packages that is equal to that many pencils? • How many students may be in one class? 20? 24? 30? 36? • How can we use either multiplication or division to express the solutions to these problems?	**Differentiated Instruction:** **English Learners:** Using sentence frames Using counters Working in pairs or small groups **Special Needs:** Working in pairs or small groups Using counters Using sentence frames Using modified numbers of students

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	Monitor students as they work, and observe the following: • What tools are students using to help them solve the problems? • How do students keep track of the steps in each problem? • Are students able to recognize a problem as a multiplication or division situation? • How do students use multiplication or division notation to record their work? • Do students deal with remainders sensibly as they answer the questions? Bring about a discussion of the various strategies used to solve this series of problems. Perhaps some students began with one strategy, then switched to another, as the problem got more complex. Perhaps some students noticed that six pencils is equal to one half dozen. In this scenario, it would be an easy stretch to also notice that two students could share one package of 12 pencils. Is there an even or odd number of students in the class? Does this affect the strategies used? Call attention to the types of tools students used to assist them in their work. Compare organizational results from using graph paper to lined or unlined paper. Ask those using cubes or counters to explain how these tools helped their thinking process. Look for students who utilized tally marks, or other methods to keep track of packages of pencils. Highlight students who created a function table or T-chart. Possible solution: 30 students need six pencils each. The total number of pencils to order is 180. We could write the equation as 30 x 6 = 180. Or we could say, "If we had 180 pencils and we gave six pencils to each student in the class, we must have 30 students." 180 ÷ 6 = 30. If we divide the pencils into groups of 12, we would see that 180 pencils is equal to 15 dozen. 180 ÷ 12 = 15 **Quick write:** How could you show the following story using either multiplication or division? "I bought a box of treats for my dog. The box contained 24 treats. I give my dog a treat 3 times a day. How many days will the box of treats last?"	**Accelerated Learners:** Using invented algorithms Finding multiple solution strategies Investigating different sized classes, and varying numbers of pencils for each student
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes	How does the mathematics change with different numbers of students in the class? Is it easier with 20 students or 24? With 30 or 36? What quantity of pencils is easier to work with? How does our thinking change as we give each student more pencils? Is it easier with 4 pencils or with 6?		

<table>
<tr><td>Unit: Division
Lesson # 8
Using Remainders</td><td colspan="2">Grade Level/Course
4th Grade</td><td>Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: cubes, counters, math journals, graph paper, calculators, array cards
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: Arrays and Shares, TERC</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to determine the most appropriate use of the remainder in division problem situations.</td><td>Language:
Students will explain how they dealt with the leftovers in a division problem situation.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td>TEACHER PROVIDES SIMPLE EXPLANATION</td><td>KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td>Remainder</td><td></td></tr>
<tr><td>Pre-teaching Considerations</td><td colspan="3">Students should have knowledge of arrays and equal groups.</td></tr>
</table>

Lesson Delivery

Instructional Methods

Check method(s) used in the lesson:

☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration**

☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge: Students should have knowledge of arrays and equal groups.

Context and Motivation:

Today's big idea is "How the remainder is explained depends on the problem situation. Today we will investigate the big idea through real life situations you may encounter. Let's look at our array cards. Find the cards that show 36 squares."

Write 36 ÷ 4 on the board.

"Here is a division problem. How do you read this? Which array would help you solve it?"

Can you think of a problem situation that you could write as 36 ÷ 4?"

Students may think of a division problem that involves sharing ("There are 36 marbles being shared by four friends. How many marbles will each friend get?") and division problems that involve grouping or measuring ("There are 36 marbles. I'm going to put four marbles in each bag. How many bags will I need?").

Chart several student responses as they are given.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Modeling

Introduce the formal division sign 4)$\overline{36}$

"This division notation is read as thirty-six divided by four, just like 36 ÷ 4. The total that is being divided shows up here as the second number, whereas when we use the ÷ the total being divided comes first."

"How would you use your calculator to solve this problem? Which keys would you use? Which number would you put in first?"

Give students a minute to explore with their calculators, then share their strategies and solutions.

Guided Inquiry

"What is another way we can divide 36 into equal groups? Look at your arrays for ideas. What is another problem situation we can use that involves dividing 36?"

Allow students a few minutes working in pairs to generate division problems that start with the total of 36. Give the following instructions:

- Write down each problem situation. (For example, "There are 36 children who are going to divide up into 6 teams for relay races. How many children will be on each team?")
- Write down the division notation for that problem using both) and ÷.
- Give the solution to each problem using both)$\overline{\quad}$ and ÷.

As students are working, monitor their progress, observing whether students are comfortable creating division problems, and whether they understand the correspondence between the problem situation and the written notation. Help students to read the notation correctly when asked. 36 ÷ 9 can be read as "36 divided into 9 groups" in a sharing situation, or "how many 9s are in 36?" in a partitioning situation.

Differentiated Instruction:

English Learners:
Using sentence frames
Using counters
Working in pairs or small groups

Special Needs:
Working in pairs or small groups
Using counters
Using sentence frames

Accelerated Learners:
Using invented algorithms
Finding multiple solution strategies
Using creativity to express the remainders in each situation

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	Ask some students to present their problems. Others can demonstrate how to express the problems using standard notation, or how they reached their solutions. Write 36 ÷ 5 on the board. Ask students to look at the problem. "What is a situation where we might have 36 divided by 5?" List student responses on the board. Then use students' situations to illustrate the problem. Discuss with the students how you would find the solution to 36 ÷ 5, and what would happen with the extra. Students may wish to express this extra as a fraction, a decimal, a remainder, or leftover amount. "Now we are going to solve a set of division problems where you cannot divide the total evenly. Your task is to decide what to do with the leftovers. Write down your reasoning and a solution for each problem. You may use any of your mathematical tools to help you." Give students the following problems to solve in pairs or small groups: 1. There are 36 people who are taking a trip in some small vans. Each van holds 8 people. How many vans will they need? 2. Eight people are going to share 36 crackers equally. How many crackers will each person get? 3. Eight people are going to share 36 balloons equally. How many balloons will each people get? 4. 36 students are going to see a movie together. Each row holds 8 people. How many rows will they fill up? 5. Eight friends raised $36 by washing people's cars. They want to share the money equally. How much money should each person get? **<u>Math Meeting</u>** Gather the students together to discuss their solutions to the division problems. Invite some students to share their solution strategies and how they expressed the remainder. Possible solutions: (Van problem) "There are 4 full vans with 4 people left. You would need 5 vans to take all the people." (Cracker problem) "Each person will get 4 crackers. Keep 4 crackers for another day." Or "Each person will get 4 ½ crackers." (Theater problem) "32 people will fit in 4 rows. 4 people will have to sit in the fifth row." Or "You fill up 4 rows and half of another row."	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes		How is the remainder expressed differently in each of these problem situations? How does the situation determine what you can do with the remainder?	

<table>
<tr><td>Unit: Division
Lesson # 9
More Using Remainders</td><td colspan="2">Grade Level/Course
4th Grade</td><td>Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: cubes, counters, math journals, graph paper, calculators, array cards
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: A Remainder of One, by Pinczes, E.; Check out book from your school library.</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to determine the most appropriate use of the remainder in division problem situations.</td><td>Language:
Students will explain how they dealt with the leftovers in a division problem situation.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="2">Academic Vocabulary
TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Quotient
Solution
Expression</td><td></td></tr>
</table>

	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		Students should have knowledge of equal groups and formal division notation.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should have knowledge of equal groups and formal division notation. **Context and Motivation:** Today's big idea is "How the remainder is explained depends on the problem situation. We are going to start today's lesson by reading a story about a very smart bug, and how he solved a problem with remainders." Read, *A Remainder of One*.	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**<u>Guided Inquiry</u>** "Yesterday we investigated many problem situations that could be solved by dividing 36 by 8. However, in each situation, the remainder had a different meaning. We are going to continue this type of investigation today. We will be looking at the same problem situation with different questions, and different solutions." Chart the division expressions, the questions, and the solutions for each of the following problems: Mrs. Ross has a postcard collection from all the vacation trips she has taken. She has 79 postcards that she wants to place into an album. She can place seven postcards on each page. How many pages will she need for all the postcards to fit? What is the division expression we will use to solve this problem? (79 ÷ 7) What is the question? (How many pages will she need?) What is the solution? (12 pages) Mrs. Ross plans to divide her 79 postcards equally among seven children. She will keep the extras. How many postcards will Mrs. Ross keep? What is the division expression we will use to solve this problem? (79 ÷ 7) What is the question? (How many postcards will Mrs. Ross keep?) What is the solution? (2 postcards) Mrs. Ross put her postcards on display at the local library for 79 days. How many full weeks is that? What is the division expression we will use to solve this problem? (79 ÷ 7) What is the question? (How many full weeks is that?) What is the solution? (11 weeks) "Hmmm! How could we have three such different answers to each of these problems when the division expression is the same each time? Talk with your elbow partner about what you think is the reason for the different answers."	**Differentiated Instruction:** **English Learners:** **Special Needs:** **Accelerated Learners:**

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	"Now we are going to solve several problem situations. I want you to think very carefully about what the question is for each problem, and how you should best express the solution for that problem. We will have a discussion after you have finished, and I want you all to be prepared to defend your answers. You may use any of your mathematical tools that you think would help your thinking." 1. Mrs. Webster wants to buy 68 postcards. They come in packages of 6. How many packages does Mrs. Webster need to buy? 2. Mr. Seng has $71 to buy postcards. Each package costs $6. How much money does he have left after buying as many packages as he can? 3. Suzanne is placing postcards into a scrapbook. She places three postcards on each page. On which page will she place postcard number 95? As students are working, circulate through the classroom, taking notes on strategies you see students using. Select students that are using strategies that are representational of many other students' work, and students that are using innovative strategies that will push others forward in their thinking. Sequence students in a way that each student's work builds upon the previous student's work. (i.e., look for students that have drawn a picture, and another that has made a table using the same numbers.) **Math Meeting** Bring the students together to share their work, discuss their solutions, and compare their strategies. Call on various students to defend their work. Possible questions: Can you explain how you selected that solution? How was the remainder expressed in this problem solution? Why did you choose to round up (or down?) in this problem? Why did you choose the remainder as the solution to this problem?	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes		How is the remainder expressed differently in each of these problem situations? How does the situation determine what you can do with the remainder?	

Unit: Division **Lesson # 10** Menu Activities	**Grade Level/Course** 4th Grade	**Duration:** 60 min. **Date:**
Common Core Standards	**4th Grade Operations and Algebraic Thinking** **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
Materials/ Resources/ Lesson Preparation	**Textbook:** 4th Grade Houghton Mifflin Intervention Activities **Mathematical Tools:** array cards **Media/Technology to be used to deepen learning:** ST Math Whole Number Multiplication and Division Module **Supplementary Materials**: Menu Activities; Houghton Mifflin Chapter 14 Math Centers, p. 298C, Chapter 15 Math Centers p. 324C	
Objectives	**Content:** Students will be able to solve division problems using visual and numerically strategies.	**Language:** Students will explain their thinking while playing division games.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒**Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary TEACHER PROVIDES SIMPLE EXPLANATION	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**

	STUDENTS FIGURE OUT THE MEANING		

Pre-teaching Considerations	Students should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings.

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☐ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge: Students should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings.
Context and Motivation:
To the Teacher: The following activities are provided so that students have independent opportunities to practice multiplication and division, while the teacher has the opportunity to meet with students who are struggling.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Students work on the Menu Activities, while the teacher works with small group of students. Students could choose from Menu Activities 1-4, however teach students new activities.

The following are **Menu Activities**:
The Game of Leftovers

In students math journals students should keep a record of Menu Activities that they complete.

Intervention Activities from Houghton Mifflin
Strategic Intervention **Book**:
Inverse Operations: NS 24-28; NS 41
Remainders: NS 40, NS 42

Benchmark Intervention **Book**:
Remainders: pp. 30, 57-58, 60
Problem Solving: p. 31

English Learners Resources **Book**:
Using Language and Solving Word Problems pp. 103-106 (for Beginning and Early Intermediate English Language Learners)
Can be either Teacher Directed or students can listen to the CD and complete the activities in small groups or pairs.

Differentiated Instruction:

English Learners:

Special Needs:

Accelerated Learners:

Lesson Reflection

Teacher Reflection Evidenced by Student Learning/ Outcomes	Call students together to debrief the Menu Activities Session. Ask students: What challenges did you find while working? Is there a learning pair who wants to share how well they worked together? Why? Which activities did you complete? How did the activity help you?

The Game of Leftovers

You need: A partner
One die
15 Color Tiles
One cup to hold the tiles
Six paper plates or 3-inch paper squares ("plates")

1. Take turns. On your turn, roll the die, take that number of paper plates or squares, and divide the tiles among them. Keep any leftover tiles.

2. Both players record the math sentence that describes what happened.

   For example: $15 \div 4 = 3$ R3

   In front of each sentence write the initial of the person who rolled the die.

3. Return the tiles on the plates to the cup before the next player takes a turn.

4. Play until all the tiles are gone. Then figure your scores by counting how many tiles each of you has. The winner is the player with the most leftovers. Add your scores to make sure that they total the 15 tiles you started with.

5. When you finish a game, look at each of your sentences with a remainder of zero (R0). Write on the class chart each sentence with R0 that isn't already posted.

Starting Menu Activities

If you set up your choices at stations, list the materials students will find at each station. Students can keep track of their choices on their own choice lists.

Choice 1: Array Game: Multiplication Pairs: directions; Array Cards, Sets A and B (1 set per pair)
Choice 2: Array Game: Count and Compare: directions; Array Cards, Sets A and B (1 per pair)
Choice 3: Array Game: Small Array/Big Array: directions; Array Cards, Sets A and B (I set per pair)
Choice 4: Hungry Ants: directions; paper to record (1 per pair)
Choice 5: The Game of Leftovers
Choice 6: Mystery Numbers

Make copies of game directions available or simply post each sheet. Students may refer to the directions when in doubt about the rules of the game. Students may choose to play using only the Array Cards in Set A, which consists of multiplication pairs with products up to 50. Then when they feel comfortable, students may include Array Cards from Set B.

Choice 1: Array Game: Multiplication Pairs
Given the dimensions of an array, students are to find the total number of squares in the array; given the total, students are to find the dimensions. As they play, students write the multiplication pairs and relationships they know and don't know on a sheet of paper.

Choice 2: Array Game: Count and Compare
Students use multiplication relationships to find the sizes of students' array cards and then determine the largest.

Choice 3: Array Game: Small Array/Big Array
Students use their array cards to make "matches" between a large array and two or more smaller arrays. Each student should write their "matches" on a sheet of paper using mathematical statements.

Choice 4: Hungry Ants:
Students explore how to group other numbers of ants.

Choice 5: The Game of Leftovers
Students explore how to work with the leftovers in a division problem.

Choice 6: Mystery Numbers
Students use number strategies and their knowledge of prime and composite to find the mystery number. They could use scratch paper or graph paper to find a solution.

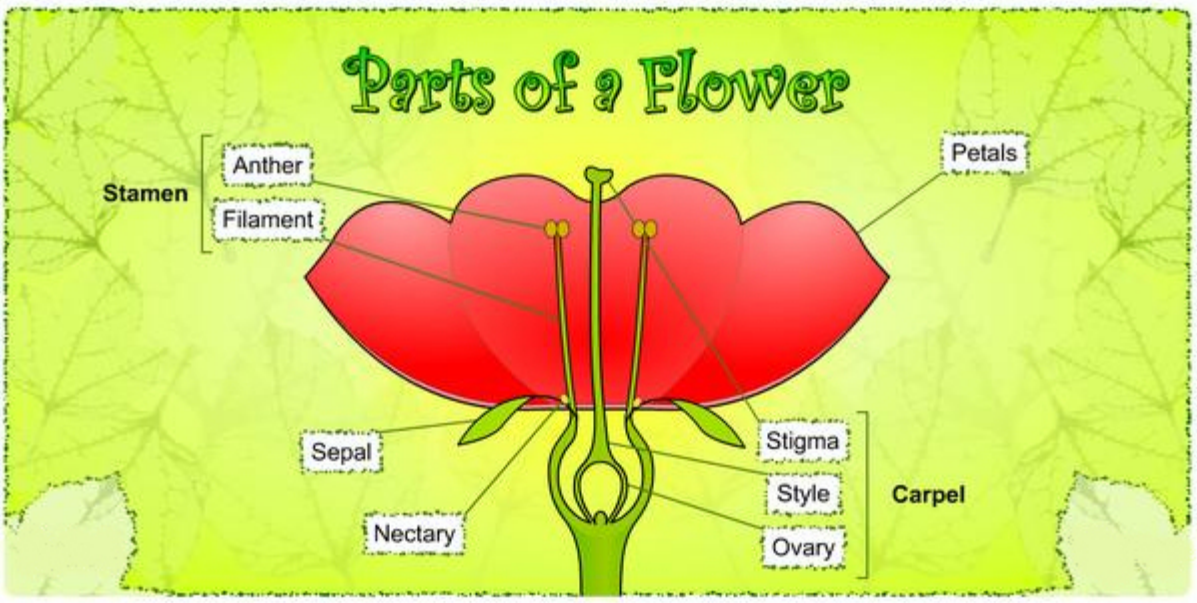
During Choice Time, circulate among the groups and observe students as they are involved with an activity, or use the time to meet with small groups of students who are having difficulty with a particular activity.
Some things you might look for are the following:

- How are students making decisions about choosing an activity and organizing their time and materials?
- Are there too many or not enough activities going on at once?

- Are students keeping track of the choices they have completed?
- How are students figuring out the total number of squares in arrays? Are they counting one by one? Counting by groups? Do they know the multiplication pairs?
- Do some students need to spend more time counting by 2s, 3s, and 6s?
- Are some students ready to add the next set of arrays (Set B) to their existing set?

<table>
<tr><td>Unit: Division
Lesson # 11
Prime & Composite Numbers</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: graphing paper, counters 20 per pair, or cubes
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to analyze given whole numbers to prove numbers are either prime or composite.</td><td>Language:
The students, given a counting number less than 100, will be able to express if it is a prime number or a composite number and why.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td></td><td>KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td>TEACHER PROVIDES SIMPLE EXPLANATION</td><td>Factor
Multiple
Prime number
Composite number</td><td></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td>Whole number
Counting number</td><td></td></tr>
</table>

Pre-teaching Considerations		Students should be able to identify odd and even numbers.
Lesson Delivery		
Instructional Methods		**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☐ **Collaboration** ☐ **Independent Practice** ☐ **Guided Inquiry** ☐ **Reflection**
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Student should be able to identify odd and even numbers. **Context and Motivation:** Today we want to prove the big idea "**Rules of divisibility are based on number patterns."** with conjectures and evidence. Ask students what they think the word divisibility means. Jot down their thoughts below the word. Tell them that we will define the words prime and composite and then prove the big idea. Referring back to their list of things that come in groups, tell students that some things come in groups and groups of groups, for example, you may buy 3 packages of pencils, while each package has 12 pencils (provide a visual- quick draw or actual objects). Show students the parts of a flower. Notice petals, sepal, anther.



Ask students to observe the following flowers:
What do you know about the flowers? What do you know or observe about their structures? How are they the same? How are they different? Create the following chart.

Daisy	Lily
Many, many petals Each daisy petal has its own sepal Daisy has many anthers Each petal is its own flower A daisy is many flowers growing tightly together.	Less petals Few sepals All its parts make one flower

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Define counting numbers for students: Counting numbers or natural numbers are 1, 2, 3, 4, ... Tell students that today we are going to use counting numbers to define what prime numbers are.
Review "Factor" with students. Factor x factor = product

Each dyad or triad of students will need 20 counters.
Ask the students to take 6 counters and arrange the 6 counters in 2 or more rows and with the same number of counters in each row.

O O O
O O O or

O O
O O
O O

Ask the students to take 5 counters and arrange the 5 counters in 2 or more rows with the same number of counters in each row. Students should notice that it cannot be done.

O O O
O O

Ask the students to take 9 counters and arrange the 9 counters in 2 or more rows with the same number of counters in each row.

O O O
O O O
O O O

Using their counters, ask the students to complete the following information for each number listed. Do the first four with them.

Number	Drawing of counters arranged in rows	What are its factors?	Prime Number	Composite Number
5	•• •••	1, 5		
6	••• •••	1, 2, 3, 6		
7	••• ••••	1, 7,		
8	•••• ••••	1, 2, 4, 8		

Guiding Questions:

- Ask students what they notice about the number 5 and 6.
- What do they notice about 6 and 8?
- What can they conjecture about prime and composite numbers so far?

Give students time to discuss this with a learning partner. Chart their responses.

Differentiated Instruction:

English Learners:
Students use manipulatives.
Students work with a partner.
Teacher provides sentence frames for class discussions.
Use of visuals

Special Needs:
Use of visuals.
Students use manipulatives.
Students work with a partner.
Is a prime number divisible by two?
Connection to real or natural life.

Accelerated Learners:
Houghton Mifflin, Enrichment 14.3- Charting Primes and Composites.

		Math Meeting Direct students to complete their charts up to 20. Bring them together for a **Math Meeting**. Ask students to put an x in the box under Prime Number if the number has two factors and even rows, and an x in the box under Composite Number if the number has uneven rows and more than two factors. Then ask students to revise their conjectures. Students should conjecture that: The prime numbers will be those numbers for which 2 or more rows with the same number of counters in each row **cannot be formed.** The composite numbers are those numbers for which 2 or more rows with the same number of counters in each row **can be formed.** Now create a t chart with yes and no. Show students a set of numbers one at a time and ask them where to put it. Allow students to place the number in the chart and tell why they placed it there: 35, 28, 40, 25, 37, 57, 33, 75, 68 "This is a prime number because _____" YES | NO _____ | _____ _____ | _____ _____ | _____	

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	***How are prime and composite numbers related to the daisy and the lily?*** *Draw a bridge map of students' responses.* What are some other conjectures can you make about the big idea "**Rules of divisibility are related to prime and composite numbers"**? Chart any new conjectures. i.e., prime numbers will signal when there are no more ways to divide. Can you think of any new examples? Bring some number examples for tomorrow. In your math journals do a quick write of what you learned, what questions you have.

Name_____ 4th Grade Division Unit

Collect data about prime and composite numbers.

	Drawing of counters	Factors	Prime Number	Composite Number
5	00 000	1,5	X	
6	000 000	1, 2, 3, 6		X
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

PRIME AND COMPOSITE NUMBERS
CONCEPT TEST

Name_____ Score_____

Determine if the number in the [ ] is prime or composite.

	Prime	Composite	
1. [43]	_____	_____	Proof: _____
2. [24]	_____	_____	Proof: _____
3. [11]	_____	_____	Proof: _____
4. [30]	_____	_____	Proof: _____
5. [21]	_____	_____	Proof: _____

PRIME AND COMPOSITE NUMBERS
CONCEPT TEST

Name_____ Score_____

Determine if the number in the [ ] is prime or composite.

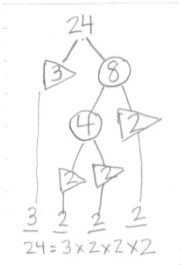
	Prime	Composite	
1. [43]	_____	_____	Proof: _____
2. [24]	_____	_____	Proof: _____
3. [11]	_____	_____	Proof: _____
4. [30]	_____	_____	Proof: _____
5. [21]	_____	_____	Proof: _____

<table>
<tr><td>Unit: Division
Lesson # 12
Factor Trees</td><td colspan="2">Grade Level/Course
4th Grade</td><td>Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools:
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module; National Library of Virtual Manipulatives Website: http://nlvm.usu.edu/en/nav/category_g_2_t_1.html;
http://pbskids.org/cyberchase/videos/#!/seasons-1-8/4 Icky's Factor ;
http://www.geneyang.com/factoring/</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to analyze a given composite number to find its prime numbers by drawing a factor tree.</td><td>Language:
The students will explain the pathways of their factor trees.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="3">Academic Vocabulary (Tier II & Tier III)</td><td>TEACHER PROVIDES SIMPLE EXPLANATION</td><td>KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td></td><td>Prime Factor
Factor Tree</td><td></td></tr>
<tr><td>STUDENTS FIGURE OUT THE MEANING</td><td></td><td></td></tr>
</table>

Pre-teaching Considerations	Students should be able to identify the factors of a number.	
Lesson Delivery		
Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☒ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum — **Lesson Opening**	**Prior Knowledge:** Students should be able to identify the factors of a number. **Context and Motivation:** Yesterday we compared prime numbers to a simple flower like the lily and composite numbers to a daisy. Ask students what prime number and composite numbers they thought of to share with the class. Create a quick chart of their ideas.  Daisy-Composite: 72  Lily- Prime: 71 Tell students we are going to construct – factor trees to find the prime and composite numbers of a number. A factor tree is a number tool you can use to find solutions to problems with fractions and to find solutions using algebra. When we study the traits of numbers we are study number theory. Number Theory helps us do more difficult mathematics. Before we draw factor trees lets create a Number Theory page in our journals. Talk students through the creation of this page. Fold the paper in half lengthwise. Fold both sides toward the centerfold. Fold the paper in half. See teacher's model in the lesson addendum.	
	Direct Instruction Today's big idea is "A number is the product of its prime factors." The tree is constructed for a particular number by looking for pairs of values which multiply together to give that number. These pairs are added as "leaves" below the original number. If a leaf is prime number, then it can be underlined as it is a prime factor. Leaves that are not prime numbers can be broken down in the same way as the original number, until all the leaves are prime numbers. Think back to the daisy and the lily. A daisy leaf can be broken down into its related factors. If you cannot break the number into factors it becomes a lily—a prime number. You are finished when you have only prime numbers! This process is called Prime Factorization	

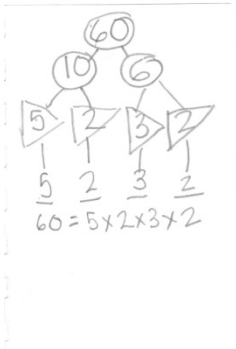
Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

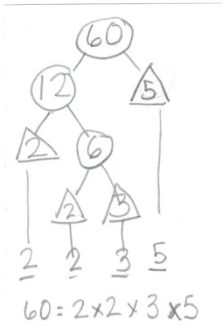
I'll try one for you. Draw a tree factor for 24



I'll draw a circle or daisy around a composite factor and a triangle around a prime factor. Tell your elbow partner why I drew a circle around some numbers and a triangle around others.

You can make different trees for one number: Draw a factor tree for 60.





Let students complete the factor trees for 36. Check student work and clear up any questions they may have.
Ask students to make a factor tree for 42 and then 50.

Guiding questions:

- How do you know when you are finished with a factor tree?
- Are the prime factors of a number always the same?
- Does the order in which the prime factors are written matters? (commutative property)

Collaboration
Now work with your partner to make a factor "Tree". Choose a composite number. Title your trees: Factor Tree: How to Use Prime Factorization.
Inside your trees create you factor trees. Check your work by multiplying all the prime factors.

Math Meeting
Ask pairs of students to share their finished products. They should use the terms prime and composite, and product. Make sure they include how they checked their work.

What does the factor tree show? (Strategic)
What is the relationship between the prime factors of a number and the number itself? (Benchmark)
When starting a factor tree what factors do you start with? (Advanced)

Differentiated Instruction:

English Learners:
Visuals and graphics
Houghton Mifflin
T.E., p. 308B
Teacher talks students through drawing a factor tree and asks students to repeat the steps.

Special Needs:
Houghton Mifflin:
Reteach 14.4

A multiplication chart may help students who need support finding factors.

Direct students to draw factor trees with appropriate numbers.

Accelerated Learners:
Houghton Mifflin:
Enrichment 14.4

Challenge students to create factor trees with larger numbers.

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

Factors	Multiples
Factors of 24	Multiples of 24
Prime Numbers	Composite Numbers
Examples of Prime Numbers	Examples of Composite Numbers

A whole number that divides a whole number without a remainder. or One of two whole numbers that multiply together to form a product.

A number that is the product of 2 factors (like skip counting)

24

1 x 24
2 x 12
3 x 8
4 x 6

Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

The first five multiples of 24 are:

24, 48, 72, 96, 120

A prime number has only 2 factors: one and itself

product and prime number → 7 = 1 x 7 ← factor, factor

A composite number has 3 or more factors.

8 = 1 x 8
8 = 2 x 4

All even numbers are composite because 2 is a factor of all

19 3 13

5 17

15 14 27

20 24

Unit: Division **Lesson # 13** Multiplication Properties	**Grade Level/Course** 4th Grade	**Duration:** 60 min. **Date:**

Common Core Standards	**4th Grade Operations and Algebraic Thinking** **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Materials/ Resources/ Lesson Preparation	**Mathematical Tools: colored pencils and plain white paper** **Media/Technology to be used to deepen learning:** ST Math Whole Number Multiplication and Division Module **Supplementary Materials**: Properties slides or power point; teacher model of journal page

Objectives	**Content:** Students will be able to describe relationships between factors and multiples.	**Language:** Students will express the relationships between numbers in the form of conjectures.

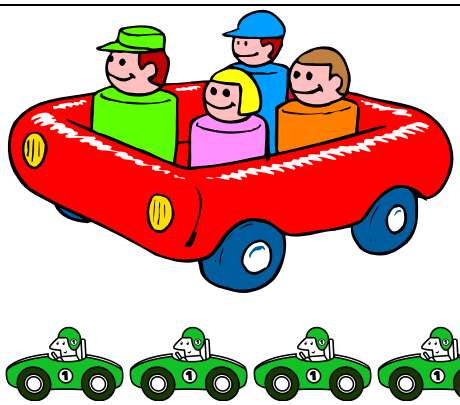
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒**Level 4: Extended Thinking**
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**

Academic Vocabulary (Tier II & Tier TEACHER PROVIDES SIMPLE EXPLANATION	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Divisible Factors Multiples Digits	Conjectures

	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		Students should have knowledge of number patterns and multiplication facts.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should have knowledge of number patterns and multiplication facts. **Context and Motivation:** "Multiplication properties demonstrate relationships between numbers. How can you prove that the properties of multiplication help with division? You have learned that there are five properties that help make solving multiplication and division problems easier. Does anyone remember what they are?" **Think-Pair-Share** Take a brief inventory of student knowledge of the properties of multiplication. You will show students slides or pictures of sets and have students infer what their relationship is. "I will show a series of slides that show relationships between numbers. Try to deduce what the relationships are."	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**Concept Attainment** Show students a slide of a set of 6 spiders.  , and 8 beetles  , a set of 6 cars  and ask them about the number of wheels, and a set of 4 dump trucks.  In their learning dyads ask them to write the expressions that go with the sets of pictures. 6 x 8, 8 x 6, 6 x 4, 4 x 6. Students should conjecture that the products are the same, no matter the order of factors. Ask them if their conjectures work for all numbers and operations. They should deduce that it is limited to addition and subtraction. In their journals have them record the Commutative Property: **The ORDER of the numbers does not change the sum of an addition problem of the product of a multiplication problem.** Does it work with all numbers?	**Differentiated Instruction:** **English Learners:** Houghton Mifflin: *English Learners Resource*: p. 70 Using sentence frames Using pictorials Working in pairs or small groups **Special Needs:** Working in pairs or small groups Using journals Using sentence frames

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**4 friends went shopping for the holidays. Each friend bought 2 boxes of cookies. Each box had 3 cookies. How many cookies did the friends buy together?**    **4 x (2 x3) = 24** **A store was selling packages of cookies. Each package has four kinds of cookies. There are two of each kind. 3 shoppers came in the bakery together. Each shopper bought a package.**   **(4 x 2) x 3 = 24** In their learning dyads ask them to write the expressions that go with the sets of pictures. Students should conjecture that the products are the same no matter how the numbers are grouped. Ask them if their conjectures work for all numbers and operations. In their journals have them record the Associative Property: **When three or more numbers are multiplied, the product is the same regardless of the order of multiplication. (Regroup)** **so, 4 x (2 x3) = (4 x 2) x 3** Show student the next slide or pictures:	**Accelerated Learners:** Houghton Mifflin: *Chapter Challenges and Investigations:* p. 33 Students explore the inverse of the associative and commutative properties.

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding



In their learning dyads ask them to write the expressions that go with the sets of pictures. Students should conjecture that the products are the same as the factor, if factor multiplied by one. Ask them if their conjectures work for all numbers and operations. For example; 4 x 1 = 4 or 1 x 4 = 4 so, 4 x 1 = 1 x 4
Just like each of us has our own identity, a number has its own identity as well. We are all unique in that every person has their own identity, which is themselves, a number has its own identity as well, itself.

In their journals have them record Multiplicative Identity Property:
The product of any number and one is that number.

Ask students what the inverse would be: 4 ÷ 1= 4 or 4 ÷ 4 = 1

Six pickle jars, but no pickles:

In their learning dyads ask them to write the expressions that go with the sets of pictures. Students should conjecture that any number multiplied by zero equals zero.

In their journals have them record The Zero Property: **The product of zero and any number is zero.**
Examples: **6 x 0 = 0; 0 x 6 = 0**

When students have completed taking notes for their journal ask them to teach each other (reciprocal teaching).

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**Practicing the Properties:** 1. Which property of multiplication is shown? $(5 \times 6) \times 3 = 5 \times (6 \times 3)$ 2. Which property of multiplication is shown? $9 \times 8 = 8 \times 9$ 3. Which property of multiplication is shown? $0 = 9 \times 0$ 4. Which property of multiplication is shown? $1 \times 5 = 5$ 5. Which equation shows the commutative property of multiplication? a. $8 \times (1 - 0) = 8 \times 1 - 8 \times 0$ b. $7 \times 8 = 8 \times 7$ c. $5 \times (3 + 9) = 5 \times 3 + 5 \times 9$ d. $8 + 8 + 8 + 8 + 8 = 5 \times 8$ 6. Which property of multiplication is shown? $6 \times 5 + 2 \times 5 = (6 + 2) \times 5$ 7. Which property of multiplication is shown? $4 \times 0 = 0$ **Reflection:** **How can you use the properties of multiplication to help with division?** The Commutative and Associative Properties help make your multiplication accurate. The properties offer different ways to solve the same problem, often in an easier way. By allowing for different opportunities to solve the problem, it is easier to check your work and find a correct product. Glue journal page into notebook.	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

<table>
<tr><td>Unit: Division
Lesson # 14
Divisibility Rules</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: index cards, colored pencils and plain white paper
Media/Technology to be used to deepen learning: ST Math Whole Number Multiplication and Division Module
Supplementary Materials: Teacher model of journal page; Write the following numbers on index cards 6025, 1230, 723, 846, 3421, 680, 975, 2963</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will be able to use divisibility rules to find factors related to a dividend.</td><td>Language:
Students will express the rules of divisibility in the form of conjectures and record them in their math journals.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☐ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒ Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
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<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
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<tr><td rowspan="2">Academic Vocabular</td><td>TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">Divisibility
Digits</td><td>Conjectures</td></tr>
</table>

	STUDENTS FIGURE OUT THE MEANING	Factors Multiples	
Pre-teaching Considerations		Students should have knowledge of number patterns and multiplication facts	

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge: Students should have knowledge of number patterns and multiplication facts

Context and Motivation:

Rules of divisibility are tools to solve division problems. (The Rules of Divisibility are listed in HM Lesson 14.2. Only teach the rules that you think are useful to your students. It will be difficult to teach all of the rules. 4th grade usually focuses on 2, 3, 5, 9, and 10)

Tell students: **Yesterday we explored how the multiplication properties are related to division. Today we will investigate rules of divisibility and practice how to use them as a tool to solve division problems. Rules of divisibility are number patterns that can be used to estimate the factor related to the dividend and divisor. Let's define divisibility.**

Write the following problems on the board and assign tables to solve for one problem.

$5\overline{)1245}$ $5\overline{)3671}$

Randomly choose student groups to share out their solutions. Ask them what they noticed about the quotients of the two problems. (one number could be divided evenly and the other had remainders.) Give students the definition of divisibility: be able to be divided with no remainder.

"Have you ever wished that you could tell if an example would divide out evenly or have a remainder before you divide?"

Tell students: "**Suppose I told you I am a math magician and can do this. I'll prove this to you by simply looking at some numbers and then telling you if they will divide evenly by 5."**

Write the following numbers on cards: 6025, 1230, 723, 846, 3421, 680, 975, 2963. Ask students to randomly choose a card and respond yes or not to its divisibility. (You are also modeling the language for students. You may want to place a sentence frame next to the T-chart so students internalize the language)

Have students look at all the numbers in the Yes column. Tell them to look for similarities in the numbers. If necessary, use guiding clues such as "focus on the digit in the ones place" or underline the last digit in the yes column. After someone has discovered the pattern, formulate a rule and write it on the board: If a number ends in the digits 0 or 5, it will always be evenly divisible by 5.

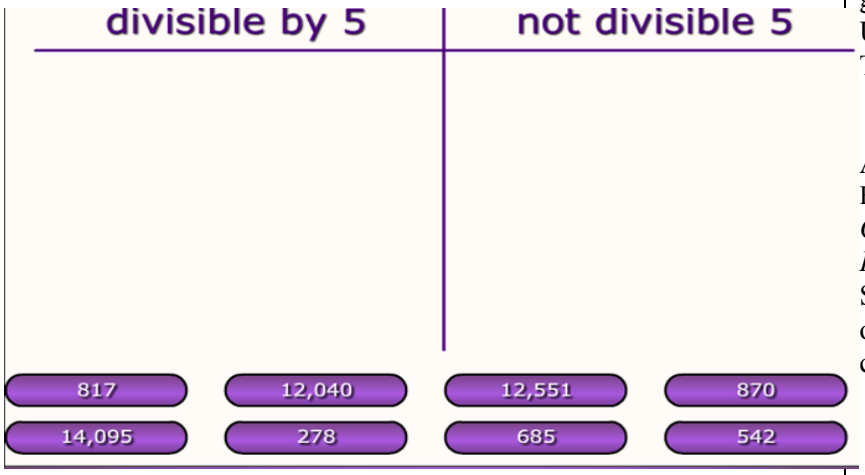
YES: 6025, 1230, 680, 975 NO: 3421 723 846 2963

Divisible by 5	
Yes	**No**

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

Concept Attainment

Show students how to fold and cut their journal page for Rules of Divisibility. It should be folded widthwise into 4 columns, and length wise into 4 columns. They should fold the end columns toward the center and then cut the front folds. (See teacher's model)
Model and write the definition of divisibility behind the first fold.
Model and write the rule for 5 behind its fold.
If you want more practice, you can do the following classification activity of sorting the following numbers into the appropriate columns on a tree map. (See teacher addendum for charts). Give students time to think. Ask them to focus on two or three numbers and chat with a partner about where their numbers should go. Randomly ask students to tell you where to write the numbers or have students write the numbers in the columns on this Tree Map.



- Explain that there are some divisibility rules for other numbers that you divide by such as 2.
- Put these numbers on the board and tell students they are all evenly divisible by 2: 62, 86, 124, 798, 2486, 7420. (Have a student verify with a calculator that all of these numbers are evenly divisible by 2.
- Have students look for similarities in these numbers. If necessary, ask guiding questions to help them discover that all of the numbers are even.
- Make up a divisibility rule for numbers that will divide evenly by 2 and write or model how to write it in their journals.

Rule: If a number ends with a 0, 2, 4, 6, or 8, then it is divisible by 2. Every even number is divisible by 2.

Differentiated Instruction:

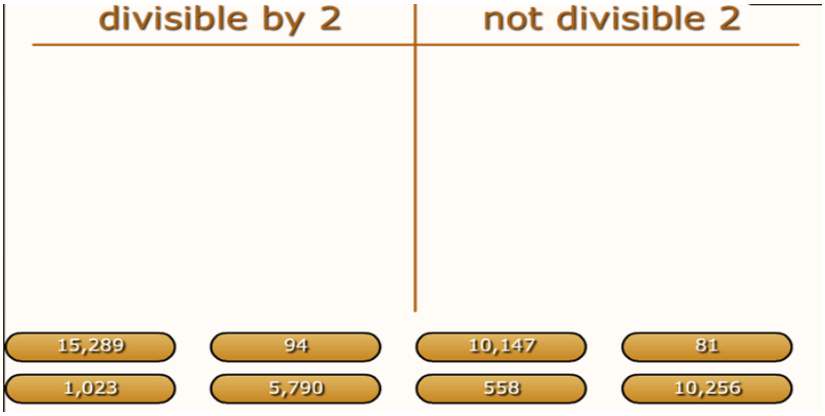
English Learners:
Divisibility Poem/Chant
Using pictorials
Working in pairs or small groups
Teacher prompts

Special Needs:
Houghton Mifflin:
Reteach: 14.2
Working in pairs or small groups
Using journals
Teacher prompts

Accelerated Learners:
Houghton Mifflin:
Chapter Challenges and Investigations: 14.2 p. 33
Students explore the inverse of the associative and commutative properties.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Sort the following numbers into the appropriate columns. (see teacher addendum for charts). Give students time to think. Ask them to focus on two or three numbers and chat with a partner about where their numbers should go. Randomly ask students to tell you where to write the numbers or have students make a tree map and write the numbers in the appropriate place.



Guided Practice

Tell students there's a trick to knowing if a number is evenly divisible by 3.

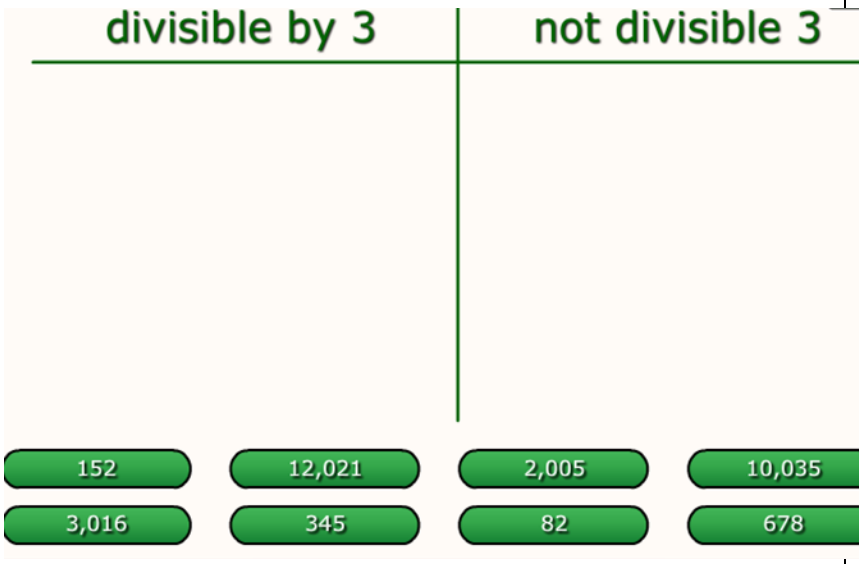
Rule: If the sum of the digits is a multiple of 3, the number is divisible by 3.

Model how to write the rule in their journals. Have them look at these numbers and tell if they are evenly divisible by 3: 405, 381, 928, 4,616

Number	Divisible?	Why?
405	Yes	4 + 0 + 5 = 9 (9 is a multiple of 3)
381	Yes	3 + 8 + 1 = 12 (12 is a multiple of 3)
928	No	9 + 2 + 8 = 19 (19 is *not* a multiple of 3)
4,616	No	4 + 6 + 1 + 6 = 17 (17 is *not* a multiple of 3)

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

Sort the following numbers into the appropriate columns. (see teacher addendum for charts). Give students time to think. Ask them to focus on two or three numbers and chat with a partner about where their numbers should go. Randomly ask students to tell you where to write the numbers or have students make a tree map and write the numbers in the appropriate places.

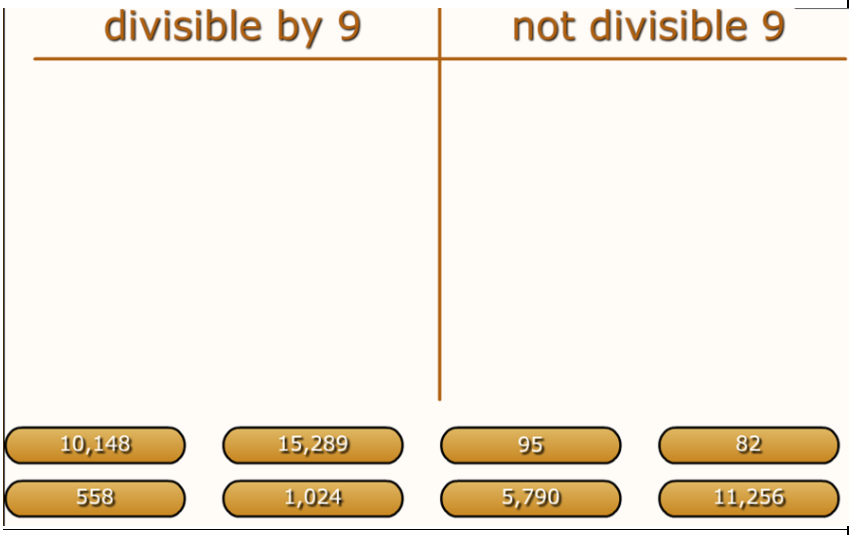


Tell students there's a trick to knowing if a number is evenly divisible by 9.
Rule: If the sum of the digits is a multiple of 9, the number is divisible by 9. Or **If the sum of the digits is divisible by 9, the number is divisible by 9.**
Model how to write it in their journals.
Have them look at these numbers and tell if they are evenly divisible by 9: 7, 686, 252, 883, 5,105

Number	Divisible?	Why?
7,686	Yes	7 + 6 + 8 + 6 = 27 (27 is a multiple of 9)
252	Yes	2 + 5 + 2 = 9 (9 is a multiple of 9)
883	No	8 + 8 + 3 = 19 (19 is *not* a multiple of 9)
5,105	No	5 + 1 + 0 + 5 = 11 (11 is *not* a multiple of 9)

Activities/Tasks/ Strategies/Technology/
Questioning/Engagement/Writing/Checking for Understanding

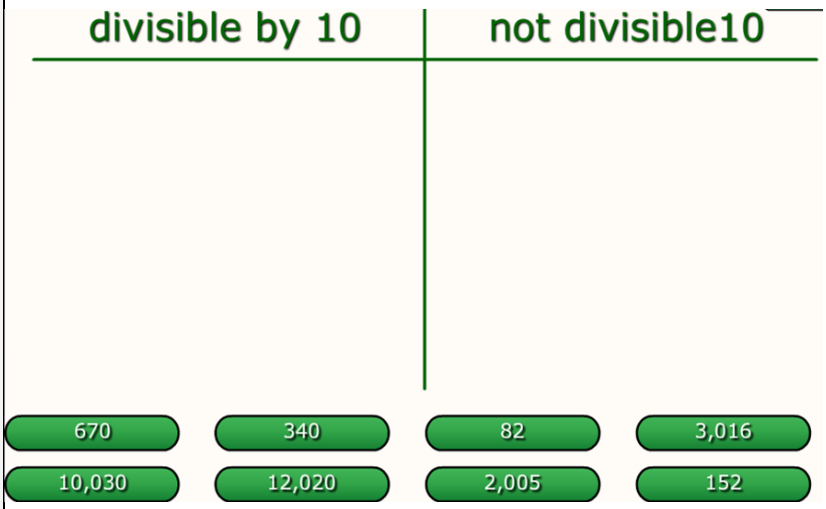
Sort the following numbers into the appropriate columns. (see teacher addendum for charts). Give students time to think. Ask them to focus on two or three numbers and chat with a partner about where their numbers should go. Randomly ask students to tell you where to write the numbers or have students write the numbers in the columns.



Tell students there's a trick to knowing if a number is evenly divisible by 10.
Rule: If the number ends with 0, the number is divisible by 10.
Give students the following numbers to think about: 880; 9,560; 312; 7,897
Model how to write it in their journals.

Number	Divisible?	Why?
880	Yes	The last digit is 0
9,560	Yes	The last digit is 0
312	No	The last digit is 2 (*not* a 0)
7,897	No	The last digit is 7 (*not* a 0)

Sort the following numbers into the appropriate columns. (see teacher addendum for charts). Give students time to think. Ask them to focus on two or three numbers and chat with a partner about where their numbers should go. Randomly ask students to tell you where to write the numbers or have students write the numbers in the columns.

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	 **Reflection:** Tell students to write the conjecture in their journals and then give an example of it. **If a given number is divisible by another number, it is a factor of the given number. Give an example:** _____. What do you understand about divisibility? What don't you understand? Glue journal page into notebook.	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Divisibility Rules

I'm #2 and I'll be your friend,
as long as an even #'s on the end.

#3 will work for me, you see,
if the sum is divisible by 3.

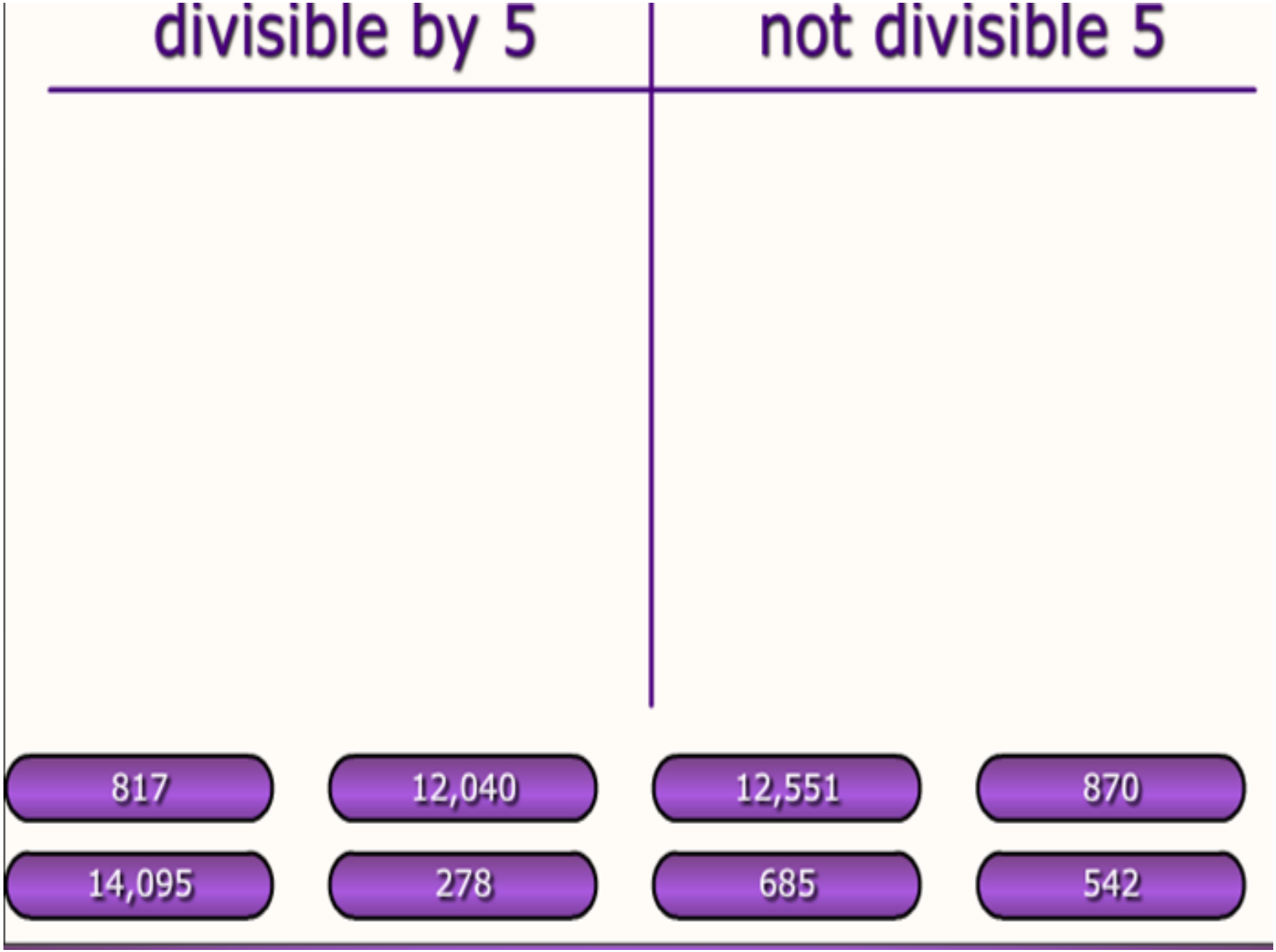
The #4 won't be such a chore,
if the last 2 are divisible by 4.

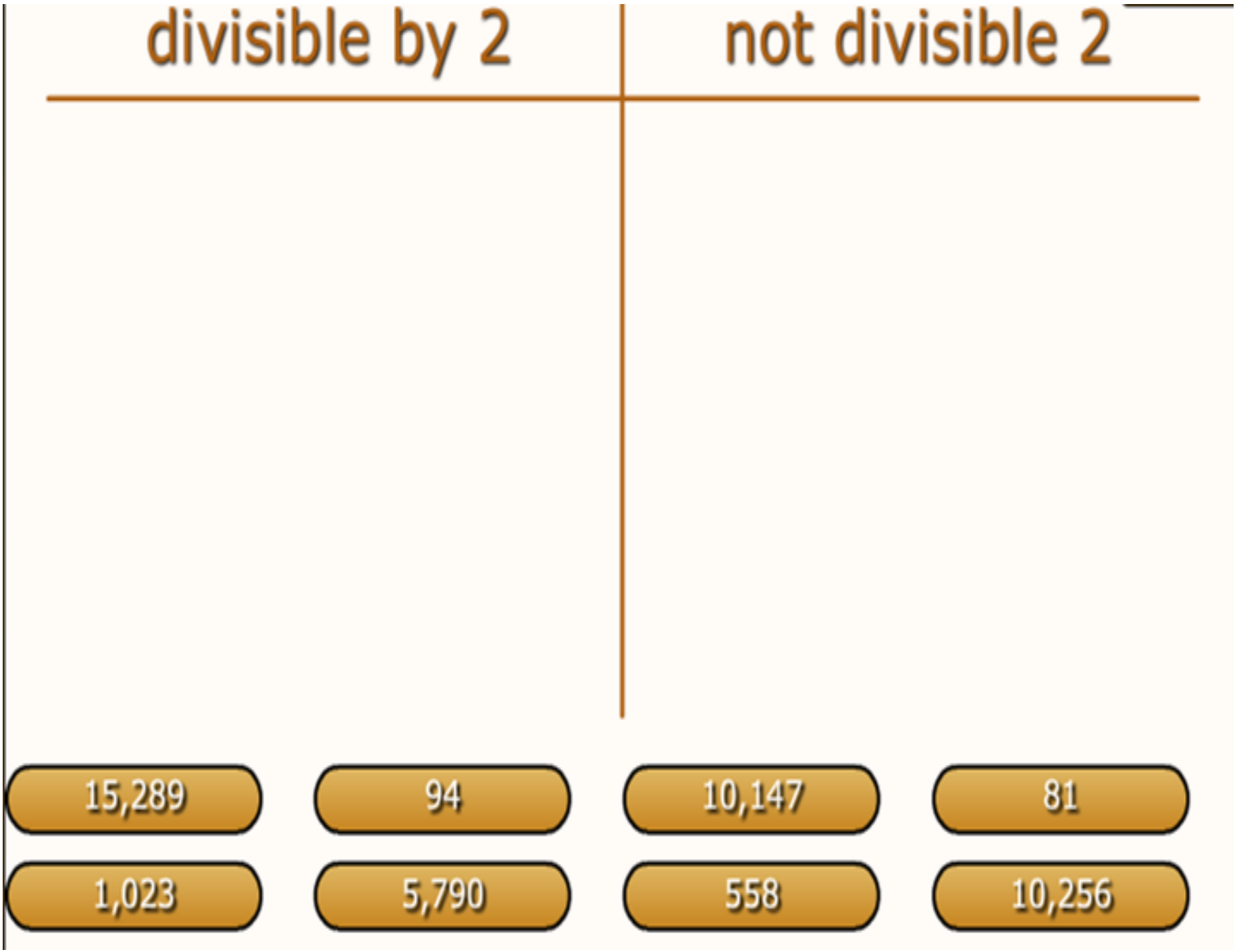
The #5 is my biggest hero,
he has to end in 5 or 0.

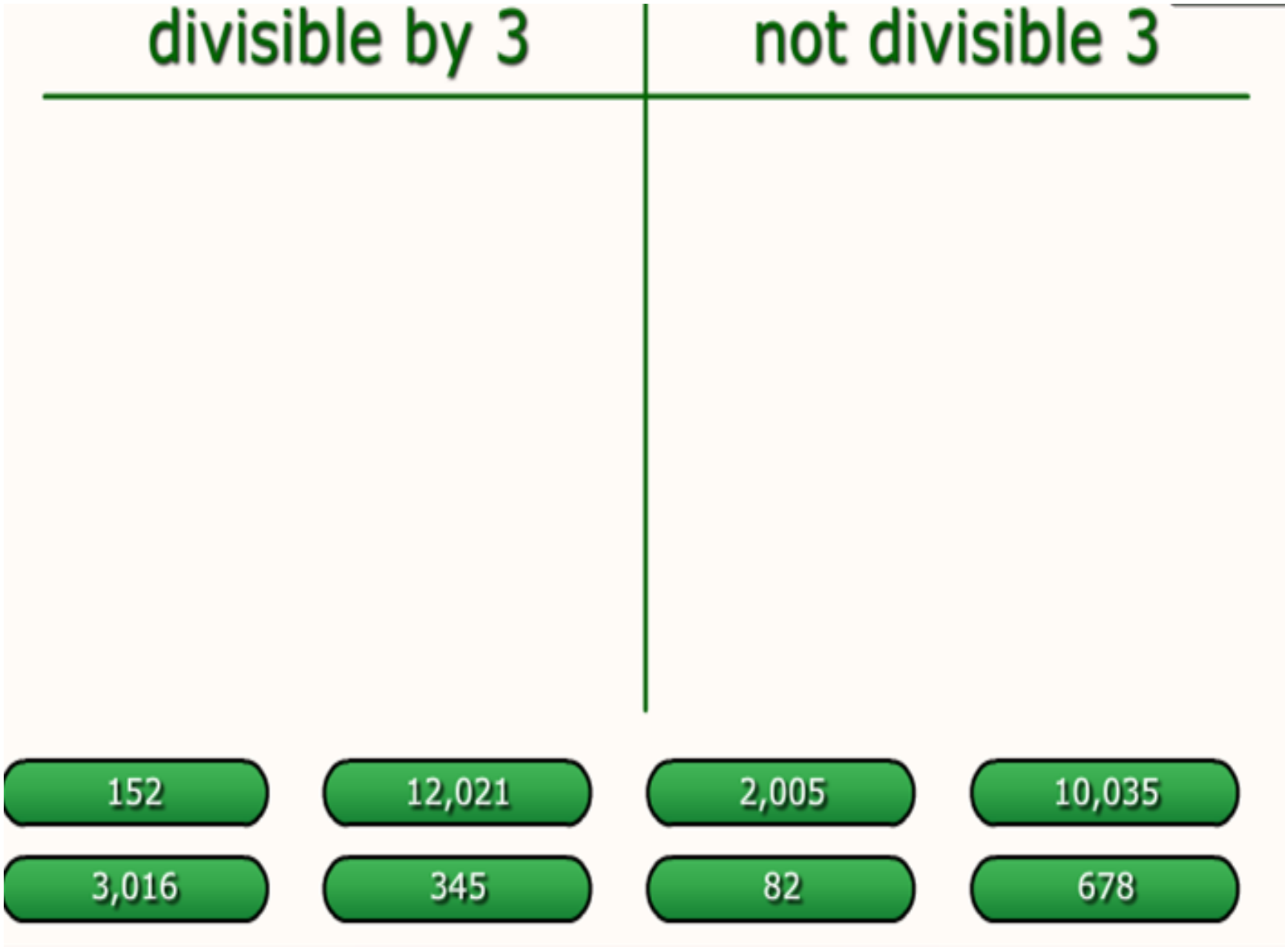
The #6 will always go into me,
as long as so does 2 and 3.

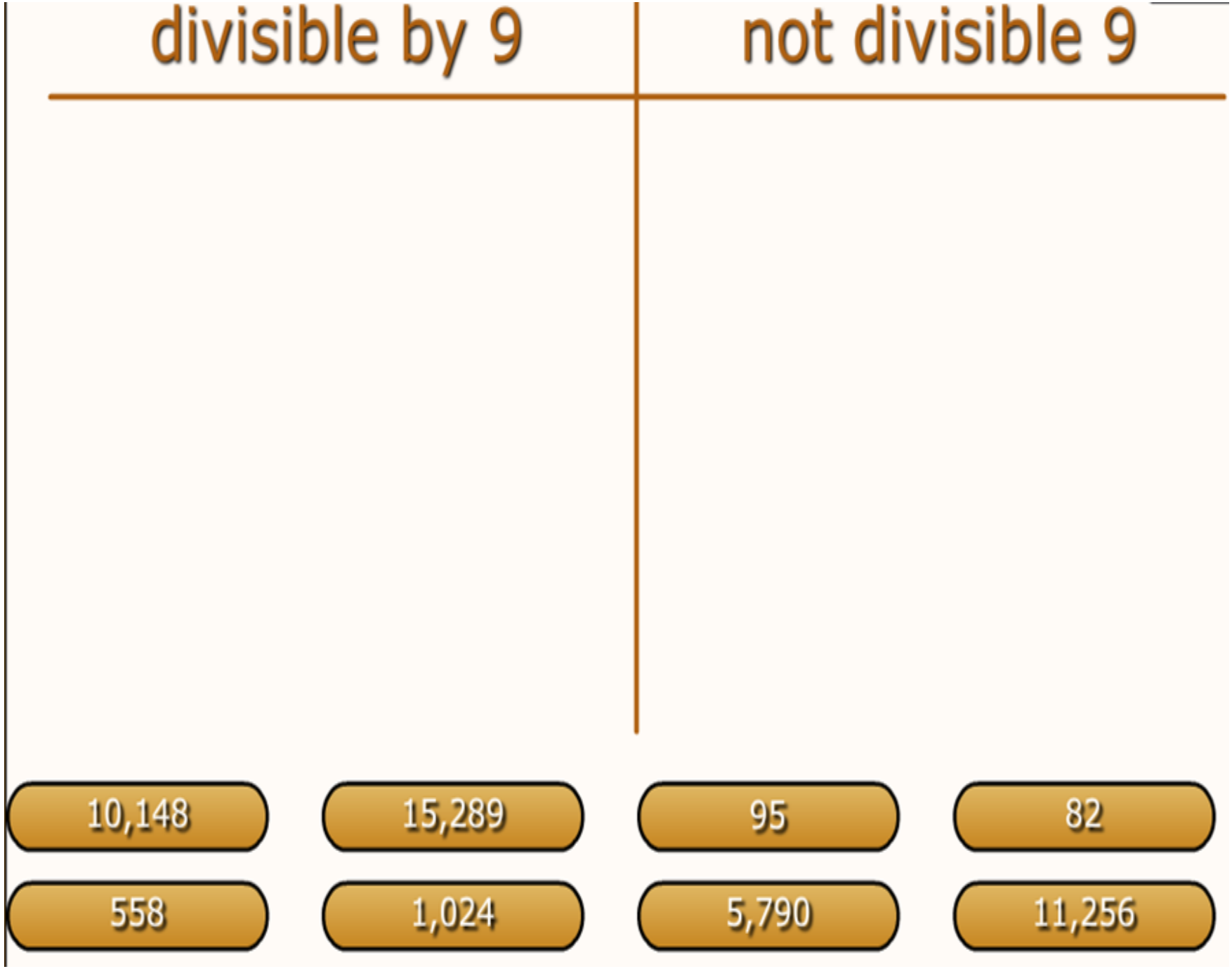
#9 will go into me just fine,
if the sum is divisible by 9.

I'm #10 and this you should know,
I always end in a big fat 0!


divisible by 5
not divisible 5
817
12,040
12,551
870
14,095
278
685
542


divisible by 2
not divisible 2
15,289
94
10,147
81
1,023
5,790
558
10,256


divisible by 3
not divisible 3
152
12,021
2,005
10,035
3,016
345
82
678


divisible by 9
not divisible 9
10,148
15,289
95
82
558
1,024
5,790
11,256

Unit: Division Lesson # 15 Menu Activities	**Grade Level/Course** 4th Grade	**Duration:** 60 min. **Date:**
Common Core Standards	**4th Grade Operations and Algebraic Thinking** **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	
Materials/ Resources/ Lesson Preparation	**Textbook:** 4th Grade Houghton Mifflin Intervention Activities **Mathematical Tools:** counters, graph paper **Media/Technology to be used to deepen learning:** ST Math Whole Number Multiplication and Division Module **Supplementary Materials**: Menu Activities, Houghton Mifflin Chapter 14 Math Centers, p. 298C, Chapter 15 Math Centers p. 324C	
Objectives	**Content:** Students will solve division problems using a variety of strategies.	**Language:** Students will explain their thinking while playing division games.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☒ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary (Tier II & Tier TEACHER PROVIDES SIMPLE EXPLANATION	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Review previous vocabulary	

	STUDENTS FIGURE OUT THE MEANING		

Pre-teaching Considerations	Students should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings.

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge: Students should be able to relate multiplication and division as inverse operations. Students should also be able to interpret a division expression and form equal groupings.
Context and Motivation:
To the Teacher: The following activities are provided so that students have independent opportunities to practice multiplication and division, and so that the teacher has opportunity to meet with students who are struggling.

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Students work on the Menu Activities, while teacher works with small group of students. Students could choose from Menu Activities 1-4, however teach students new activities.

<u>The following are **Menu Activities**</u>:
Houghton Mifflin Chapter 14 Math Centers, p. 298C, Chapter 15 Math Centers p. 324C
In students math journals students should keep a record of Menu Activities that they complete.

<u>Intervention Activities from Houghton Mifflin</u>
***Strategic Intervention* Book**:
Properties: NS 19-20
Problem Solving: NS 29-31, NS 39
Multiplying by 10s: NS 32
Divisibility Rules: NS 37-38
Factors: NS 45

***Benchmark Intervention* Book**:
Properties and Divisibility Rules: pp. 28-29
Multiples of Ten: p. 59
Model Division: p. 61-70

***English Learners Resources* Book**:
Using Language and Solving Word Problems pp. 107-114 (for Beginning and Early Intermediate English Language Learners)
Can be either Teacher Directed or students can listen to the CD and complete the activities in small groups or pairs.

Differentiated Instruction:

English Learners:

Special Needs:

Accelerated Learners:

		<u>Closing/Debrief</u> Call students together to debrief the Menu Activities Session. Ask students: What challenges did you find while working? Is there a learning pair who wants to share how well they worked together? Why? Which activities did you complete? How did the activity help you?	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

8 MYSTERY NUMBERS

"I'm thinking of a single-digit whole number..."

1) The number is odd.
2) The number has the greatest value of any single-digit prime number.

9 MYSTERY NUMBERS

"I'm thinking of a single-digit whole number..."

1) The number is even.
2) The number is a perfect cube.

10 MYSTERY NUMBERS

"I'm thinking of a single-digit whole number..."

1) The number is odd.
2) The number is composite.

20 Two-Digit Mystery Numbers

Answer Key On Back

1 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) Both digits are odd and prime.
2) The ones digit is four more than the tens digit.

2 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit is neither prime nor composite and has value.
2) The ones digit has twice the value of the tens digit.

3 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) Both digits are odd and prime and less than six.
2) The tens digit is two more than the ones digit.

4 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit is a perfect square and has value greater than the ones digit.
2) The ones digit is the cube of 2.

5

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit and the ones digit are consecutive even numbers.
2) The tens digit has half the value of the ones digit.

6

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The ones digit has half the value of the tens digit.
2) The tens digit is the square of an even number.

7

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit is a factor of 10 and 15 and is greater than 1.
2) The product of the digits is 0.

8

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The ones digit has the greatest value of any single-digit odd number that is also prime.
2) The tens digit has the same value as its own perfect square.

9

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit is even and a perfect cube.
2) The ones digit is neither prime nor composite and has value.

10

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit and the ones digit are consecutive whole numbers.
2) The tens digit is neither prime nor composite and has value.

11

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The ones digit is one greater than the tens digit.
2) Both digits are prime and factors of 6.

12

MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit has value four times that of the only even number that is also prime.
2) The ones digit has value equal to the value of the tens digit.

13 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit has one-third the value of the ones digit.
2) Both digits are odd and have value greater than 1.

14 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit has the least value of any prime number that is also odd.
2) The ones digit has the least value of any even number.

15 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit is neither prime nor composite and has value.
2) The ones digit is neither prime nor composite and has no value.

16 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) Both digits are perfect squares.
2) The sum of the digits is 18.

17 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit has the least value of any number that is odd and prime.
2) The ones digit has four times the value of a number that is neither prime nor composite and has value.

18 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The ones digit is a placeholder.
2) The tens digit is prime and even.

19 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit is the only single-digit number that is odd and composite.
2) The ones digit is the only number that is even and prime.

20 MYSTERY NUMBERS

"I'm thinking of a two-digit whole number..."

1) The tens digit is the first in the series of prime numbers that are also odd.
2) The ones digit has one-third the value of the only single-digit number that is odd and composite.

Unit: Division **Lesson # 16** Dividing Larger Numbers	**Grade Level/Course** 4th Grade	**Duration:** 60 min. **Date:**

Common Core Standards	**4th Grade Operations and Algebraic Thinking** **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** index cards, adding machine tape or sentence strips, calculators, 300 charts, math journals **Media/Technology to be used to deepen learning:** ST Math Whole Number Multiplication and Division Module **Supplementary Materials**: *Packages and Groups*, TERC

Objectives	**Content:** Students will use multiples of 10, 100, and 1,000 to solve division problems based on larger numbers.	**Language:** Students will explain how using multiples of 10 help them to apply division in problem situations using larger numbers.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	

Academic Vocabulary (TEACHER PROVIDES SIMPLE EXPLANATION)	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Multiples	

	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		Students should know various strategies for modeling division.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☒ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should know various strategies for modeling division. **Context and Motivation:** Today's big idea is "Division is used to solve problems in daily life." "Today we will encounter some real life problems using larger numbers that can be solved by using division. We will think about all the strategies we have been using, and think about ways to apply those strategies to larger numbers." Just as we can skip count by 3s, 5s, or 6s, and find the multiples of those numbers, we can also skip count by larger numbers like 20s, 25s, 30s or 40s and find the multiples of those numbers. We can use tools like cubes, a 300 chart, or a calculator to help us skip count by larger numbers. Let's make a list of the multiples of 30. Chart the multiples of 30 on the board, as students call them out. (30, 60, 90, 120…) Then ask students to continue the list on their own. After a few minutes, call students together to discuss strategies for finding the multiples of 30. Some students might use a calculator and push the following sequence of buttons: 30 + = = =. Others might push the buttons: 30 + 30 + 30… Some might mark the multiples on a 300 chart. Some might use a T-chart and list the multiples. Discuss these various strategies, and how they are related. Count around the class by 30s, and ask each student to write their number in their journal, so they don't forget it. Also add these numbers to your list on the board. Ask students about the patterns they see in the multiples of 30. If no one mentions the relationship to the multiples of 3, make sure that this is brought up. "Now we are going to predict my height, using the multiples of 30. How tall do you think I am, using an index card to measure, and counting by 30s?" Give students a moment to think, then take predictions. Find a spot on the wall where you can stand comfortably, and there is a clear path from the floor to the top of your head. Have students help to measure your height, using a strip of adding machine tape, or several sentence strips taped together. Tape the strip to the wall vertically. Start with the student who has the number 30, then proceed with 60, 90, 120, etc. Mark off the height of one index card (horizontally) starting from the floor and have the student write his/her number in the space. When the numbers reach knee height, stop to revise predictions. "Hmm, we are now at (180). Let's read the multiples so far. What is the pattern? What do you think? Could your prediction still be accurate? Would you like to revise your prediction? What number do you think will be on the card that is as high as the top of my head?"	

		Revise predictions at regular intervals (waist-height, shoulder-height, etc.**)** **"**What is a possible number we could have for my final height? Could it be 584? Why not? Let's think about which estimates are possible, and which ones don't fit the pattern? That's right, it has to be a multiple of 30." Continue adding numbers until you reach the end of the adding machine tape (the top of your head). (The final product will resemble a measuring tape, with multiples of 30 written in the spaces between lines.) Celebrate the closest prediction. "Our final number is (630). How many multiples of 30 is that? How can we find out, without counting all the numbers? Write ___ x 30 = _____ on the board. How many 30s are there in 630?" Give students time to devise strategies for solving the problem, then share out. Compare solutions.	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**<u>Guided Inquiry</u>** "Now you and your partner will have a chance to build your own multiple towers that are as tall as you are. Choose a number to skip count, using numbers from this list: 15, 18, 20, 24, 32, 36, 40, 45, 55, 60, 64, 70, 72, 75, or 82. You can use any of your mathematical tools to help you build your list of multiples. When you have listed at least 12 multiples, you may begin to build your towers. Take turns cutting a strip of adding machine tape as tall as the other one. You may lie down on the floor or stand up to the wall. Then take one index card, and carefully mark off the width of one card all the way from one end of the adding machine tape to the other end. In the spaces between the lines, start listing your multiples. Predict what number you might end with." Circulate around the classroom as students work, lending assistance as necessary. Document the various multiples chosen, and the accuracy of multiples listed. Ask pairs to check their work before proceeding to the towers. In addition, observe the following: • How are students using mathematical tools to assist their work? • Are students recognizing and using number patterns to build their multiple towers? • What strategies are students using to practice difficult number combinations? • Where are students getting stuck? How are they getting past these difficult areas?	**Differentiated Instruction:** **English Learners:** Working with partners Using hands-on materials **Special Needs:** Select from appropriate multiples Working with partners Using hands-on materials **Accelerated Learners:** Select from more complex multiples Answer more advanced questions

		Reflection: As students complete their towers, give them these reflective questions to answer about their multiple towers: • What number did you use to build your multiple tower? • Did your tower include 100, or any multiples of 100 (200, 300, etc.)? • If you kept building your tower until it reached the next multiple of 100, how many numbers would be in your tower? • What patterns do you notice in your tower? Call students together to share their results. **Closing questions:** • How can you express the final number on your multiple tower and the multiple you are counting by as a division expression? What will the solution tell you? (how many multiples were used to build your tower) • How did this activity help us with multiplication and division of larger numbers?	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

Unit: Division **Lesson # 17** Using Patterns	**Grade Level/Course** 4th Grade	**Duration:** 60 min. **Date:**

Common Core Standards	**4th Grade Operations and Algebraic Thinking** **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** tools of students' choice: have available counters, tiles, blocks **Media/Technology to be used to deepen learning:** ST Math Whole Number Multiplication and Division Module **Supplementary Materials**: Math Story excerpts: *How Much, How Many, How Far, How Heavy, How Long, How Tall is 1000?; Great Estimations (Lesson slides will be provided)* Math Journal pages provided.

Objectives	**Content:**	**Language:**
	Students will be able to use basic division facts and patterns of zeros to divide mentally and solve division problems with larger numbers.	Students will be able to verbally and in writing explain the process they used to solve a division problem.

Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**

Academic Vocabulary (TEACHER PROVIDES SIMPLE EXPLANATION)	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Divisor Dividend Quotient	

	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		Students should have knowledge of division by grouping or algorithm, multiplication and division facts, and understanding of place value.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☐ **Collaboration** ☐ **Independent Practice** ☐ **Guided Inquiry** ☐ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should have knowledge of division by grouping or algorithm, multiplication and division facts, and understanding of place value. **Context and Motivation:** Start with making estimation pictures. Ask students to estimate using numbers with zeros the quantity they see. Each photo will increase the amount of to be estimated. For each picture ask students to divide among their table groups; i.e. We see 80 ___ cherries. If we divide them among ourselves we will each receive ___. Ask students how they found the quotient. 80 - 10; 70 -10; 60 - 10; 50 -10; 40 - 10; 30 – 10; 20 divided by 6 and there were 2 left over. Or other groups may estimate more and divide by four. Compare strategies and focus on the patterns of use of zero. $120 \div 4 \longrightarrow 100 \div 4 = 25$ $20 \div 4 = 5$ $25 + 5 = 30$	
	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	**<u>White Board Discussion on Patterns of Zero</u>** White board discussions should move at a good pace. For today's exercises tell students they do not have to write the number sentences—they only need to write their answer and be prepared to justify it. Show students the following sequence: $42 \div 7 = 6$ $420 \div 7 = 60$ $4{,}200 \div 7 = 600$ Ask students: **What do you notice about the pattern of zeros? How does this remind you of multiplying with zeros?** First ask students to discuss their thinking with a learning partner. Then choose partners to share. Show students the following sequence of problems. Ask students to discuss their thinking with a learning partner. When they know their answers have them record it on a white board. At a signal ask students to hold up their boards so that you could see their thinking. Ask students to retell their reasoning for writing 600. $48 \div 8 = 6$ $480 \div 8 = 60$ $4{,}800 \div 8 = ?$	

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Show students the following sequence of problems. Ask students to discuss their thinking with a learning partner. When they know their answers have them record it on a white board. At a signal ask students to hold up their boards so that you could see their thinking. Ask students to retell their reasoning for writing 30 and 300.

$21 \div 7 = 3$
$210 \div 7 = ?$
$2{,}100 \div 7 = ?$

Now mix up the discussion and check for precision.
$4{,}500 \div 9 = ?$
Write the following problems for students to solve.
$3{,}200 \div 4 = n$
$420 \div 6 = d$
$1{,}400 \div k = 700$
$m \div 5 = 90$
Discuss the use of variables.

Problem Solving with Larger Numbers

Ask students to write an expression for the problem and then solve it. These are journal problems. The class may not work out all the problems. What problems are not completed, students may do for homework.

The Delmar family collected pennies. When the jar was full, Mrs. Delmar gave the pennies to her three sons. They counted 1,500 pennies and shared them equally. How many pennies did each boy get? *(500 dimes each*)

Sela has 6 times as many coins now as she had 4 months ago. If Sela has 240 coins now, how many coins did she have 4 months ago? *(40 coins)*

What about 1000 french fries? Even if you loved french fries, 1000 would be too much for one person. You could share them. A single serving has about 40 fries. How many friends would 1000 french fries feed? *(25)*

Chip collected 289 dimes. Sue collected 191 dimes. They divided all their dimes into 8 stacks. If each stack had an equal number of dimes, how many dimes were in each stack? *(60 dimes)*

Robby sees a rare 1937 penny. The cost is $210. If he saves $3 every week, will Robby have enough money to buy the coin after one year? *(No, It will take him 70 weeks to save the money (210 ÷ ÷ 3 = 70). There are only 52 weeks in a year.)*

Differentiated Instruction:

English Learners:
Houghton Mifflin:
Universal Access p.207B
Using pictorials
Working in pairs or small groups
Teacher prompts

Special Needs:
Houghton Mifflin:
Reteach 12.4

Working in pairs or small groups
Using journals
Teacher prompts

Accelerated Learners:
Houghton Mifflin:
Chapter Challenges and Investigations: Chapter 12, p. 71
Students explore the inverse of the associative and commutative properties.

		<u>Math Meeting</u> Ask a few students to share their solutions with the class. Make sure students are using academic language with coherence. Provide prompts for students if necessary. How did noticing the patterns of zeros help you solve the division problems?	
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

What about 1000 french fries? Even if you loved fried, 1000 would be too much for one person. You could share them. A single serving has about 40 fries. How many friends would 1000 french fries feed?	The Delmar family collected pennies. When the jar was full, Mrs. Delmar gave the pennies to her three sons. They counted 1,500 pennies and shared them equally. How many pennies did each boy get?
Chip collected 289 dimes. Sue collected 191 dimes. They divided all their dimes into 8 stacks. If each stack had an equal number of dimes, how many dimes were in each stack?	Sela has 6 times as many coins now as she had 4 months ago. If Sela has 240 coins now, how many coins did she have 4 months ago?

	Problem Solving with Division and Patterns of Zero
	Robby sees a rare 1937 penny. The cost is $210. If he saves $3 every week, will Robby have enough money to buy the coin after one year?

Unit: Division **Lesson # 18** Finding an Average	**Grade Level/Course** 4th Grade	**Duration:** 60 min. **Date:**

Common Core Standards	**4th Grade Operations and Algebraic Thinking** **Gain familiarity with factors and multiples.** 4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **4th Grade Number and Operations in Base Ten** **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Materials/ Resources/ Lesson Preparation	**Mathematical Tools:** calculators, math journals **Media/Technology to be used to deepen learning:** ST Math Whole Number Multiplication and Division Module

Objectives	**Content:** Students will compute averages for larger numbers.	**Language:** Students will explain how to find an average using larger numbers.
Depth of Knowledge Level	☐ **Level 1: Recall** ☒ **Level 2: Skill/Concept** ☒ **Level 3: Strategic Thinking** ☐ **Level 4: Extended Thinking**	
Standards for Mathematical Practice	☒ **1. Make sense of problems and persevere in solving them.** ☐ **2. Reason abstractly and quantitatively.** ☐ **3. Construct viable arguments and critique the reasoning of others.** ☒ **4. Model with mathematics.** ☐ **5. Use appropriate tools strategically** ☐ **6. Attend to precision.** ☐ **7. Look for and make use of structure.** ☒ **8. Look for and express regularity in repeated reasoning.**	
Common Core Instructional Shifts in Mathematics	☒ **Focus on the Standards** ☒ **Coherence within and across grade levels** ☒ **Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)**	
Academic Vocabulary TEACHER PROVIDES SIMPLE EXPLANATION	**KEY WORDS ESSENTIAL TO UNDERSTANDING**	**WORDS WORTH KNOWING**
	Average	

	STUDENTS FIGURE OUT THE MEANING		
Pre-teaching Considerations		Students should know addition of multiple addends. They should have several efficient division strategies.	
Lesson Delivery			
Instructional Methods		**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☒ **Collaboration** ☐ **Independent Practice** ☒ **Guided Inquiry** ☒ **Reflection**	
Lesson Continuum	**Lesson Opening**	**Prior Knowledge:** Students should know addition of multiple addends and division strategies. **Context and Motivation:** Today's big idea is "Division is used to solve problems in daily life." "Today we will investigate some situations in which we will need to find the average of some larger numbers." At Moorpark Elementary School, there are four fourth grade classes. They have the following number of students: 24, 25, 20, and 23. The fourth grade students have been invited to a play. However, the students must be evenly divided in order to be admitted to the theater. How many students will enter the theater in each group? Take a few minutes to think about this problem. How can we be sure that there are exactly the same number of students in each group? Ask students to share their strategies. Some may move 2 students from the class of 25, and 1 student from the class of 24, and add all three to the class of 20, to make 23 in each class. Some may add all the numbers and divide by four. Discuss how these strategies would lead to the same solution. Mathematicians use many different words to mean the average, or the one number that can be considered typical out of a list of numbers. This may be the mean, median, or mode. Today, we will develop different strategies to determine the mean of a group of numbers.	
		Guided Inquiry "A zookeeper has to know how much food to order for each of his animals. He kept a chart of the food the elephants ate each day for a week. Now he needs to know the average number of pounds eaten each day. Look at this table, and answer the questions related to the information."	

Food eaten per day in pounds		
	Male elephant	Female elephant
Monday	135 pounds	125 pounds
Tuesday	125 pounds	130 pounds
Wednesday	125 pounds	115 pounds
Thursday	120 pounds	120 pounds
Friday	130 pounds	125 pounds

	Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding	1. What was the average amount of food eaten by the male elephant each day? 2. How much greater was the average amount of food eaten by the male elephant than the female elephant? "Work together in pairs to find the answer to each question. Keep track of your work in your math journals. What strategies might you try?" Circulate among the students and notice how they attack the problems. Do they know how to determine an average in the traditional manner? Do they use innovative methods? Call students together to share their strategies. Look for connections between their methods. "Now let's work on a problem where we know the average, but we don't know one of the addends." There are three tigers at the zoo. One weighs 259 pounds. Another weighs 326 pounds. Their average weight is 294 pounds. How can we determine the weight of the third tiger? Allow students to work a few minutes, then review strategies. "Sometimes when we are asked to find averages, the results are not reasonable. Let's look at this problem:" The aquarium has 310 fish, 60 sea animals, and 20 sea birds. What is the average number of each type of sea creature? Allow students to work a few minutes, then ask, "Why would the average in this case not be reasonable? There are many more fish than either of the other kinds of sea creatures. If the numbers are too different, the average doesn't make sense." Review the following reflective questions with your students. **<u>Reflection:</u>** • What are some situations where we might want to determine an average? • How can we make sure everyone has an equal share? • What is an efficient strategy to determine an average?	**Differentiated Instruction:** **English Learners:** Working with partners Using hands-on materials **Special Needs:** Working with partners Using hands-on materials **Accelerated Learners:** Answer more advanced questions
Lesson Reflection			
Teacher Reflection Evidenced by Student Learning/ Outcomes			

<table>
<tr><td>Unit: Division
Unit Assessment</td><td>Grade Level/Course
4th Grade</td><td colspan="2">Duration: 60 min.
Date:</td></tr>
<tr><td>Common Core Standards</td><td colspan="3">4th Grade Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4th Grade Number and Operations in Base Ten
Use place value understanding and properties of operations to perform multi-digit arithmetic.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td></tr>
<tr><td>Materials/ Resources/ Lesson Preparation</td><td colspan="3">Mathematical Tools: Performance-Based Task
Supplementary Materials: End of Unit DivisionTest</td></tr>
<tr><td>Objectives</td><td colspan="2">Content:
Students will apply division concepts and skills through solving problems in the unit test.</td><td>Language:
Students will create and record a situation to fit a given division expression and solve for the expression.</td></tr>
<tr><td>Depth of Knowledge Level</td><td colspan="3">☒ Level 1: Recall ☒ Level 2: Skill/Concept
☒ Level 3: Strategic Thinking ☒Level 4: Extended Thinking</td></tr>
<tr><td>Standards for Mathematical Practice</td><td colspan="3">☒ 1. Make sense of problems and persevere in solving them.
☐ 2. Reason abstractly and quantitatively.
☐ 3. Construct viable arguments and critique the reasoning of others.
☒ 4. Model with mathematics.
☐ 5. Use appropriate tools strategically
☐ 6. Attend to precision.
☐ 7. Look for and make use of structure.
☒ 8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Common Core Instructional Shifts in Mathematics</td><td colspan="3">☒ Focus on the Standards
☒ Coherence within and across grade levels
☒ Rigor (Balance of conceptual understanding, procedural skill & fluency, and application of skills)</td></tr>
<tr><td rowspan="2">Academic Vocabulary
TEACHER PROVIDES SIMPLE EXPLANATION</td><td colspan="2">KEY WORDS ESSENTIAL TO UNDERSTANDING</td><td>WORDS WORTH KNOWING</td></tr>
<tr><td colspan="2">No new vocabulary</td><td></td></tr>
</table>

	STUDENTS FIGURE OUT THE MEANING		

Pre-teaching Considerations	Final unit assessment—Students should have gained all necessary skills.

Lesson Delivery

Instructional Methods	**Check method(s) used in the lesson:** ☐ **Modeling** ☐ **Guided Practice** ☐ **Collaboration** ☒ **Independent Practice** ☐ **Guided Inquiry** ☒ **Reflection**

Lesson Continuum

Lesson Opening

Prior Knowledge:
Context and Motivation:
Tell students: "Today you are going to show how much you have learned about division. There is nothing on the written test that you have not already learned. When you finish the End of Unit Test, take a copy of the math task."
Math Task: Students should work alone on this problem. They write a situation that reflects the division problem they wrote.

"Think of a situation describing the following problem: $287 \div 14 =$
Write a story problem and then solve it. As you solve the problem, record each step of your work so someone looking at your work would understand your thinking."

Activities/Tasks/ Strategies/Technology/ Questioning/Engagement/Writing/Checking for Understanding

Students will take the End of Unit Division Test.

Upon completion of the written test, students will complete the performance task:
"Think of a situation describing the following problem $297 \div 14 =$. Write the story problem and then solve it. As you solve the problem, record each step of your work so someone looking at your work would understand your thinking."

Scoring Rubric:

	4 Exemplary	3 Adequate	2 Somewhat	1 Minimal
Creates appropriate problem situation	A creative problem situation	The problem fits the situation	Problem rambles or is hard to follow	The problem is nonsensical
Records Steps	Detailed description of all steps	All steps are listed in sequence	3 steps listed, or out of sequence	1 or 2 steps listed
Deals with remainders	The precise use of the remainder, with explanation	Remainder used correctly without explanation	Remainder is listed as R ___	Remainder is not mentioned, ignored
Uses efficient strategies	More than one strategy is evident	Uses one strategy completely	False start or incomplete strategies	Counting, drawing relied upon to solve

Differentiated Instruction:

English Learners:
Have students draw a situation that describes the problem.
Have manipulatives available for students.

Special Needs:
Have students draw a situation that describes the problem.
Have manipulatives available for students.

Accelerated Learners:
Students can work on menu activities after they finish the performance task.

Lesson Reflection	
Teacher Reflection Evidenced by Student Learning/ Outcomes	

Fourth Grade End of Unit Division Test

Name _____

Work each problem in the space provided.

Circle the correct answer for each problem

1. Ms. Cortez is passing out bookmarks to a group of students in her class. She has a total of 80 bookmarks, and there are 6 students in the group. If she gives each student the same number of bookmarks, how many bookmarks will she have left over? A 1 B 2 C 4 D 5	2. A truck driver drives 2800 miles each week. How much does the truck driver drive each day, if he drives the same number of miles each day? A 280 miles B 400 miles C 700 miles D 1400 miles
3. Isabella has 6 times as many pennies as she had 4 months ago. If Isabella has 420 pennies now, how many pennies did she have 4 months ago? A 70 B 88 C 90 D 109	4. A long roller coaster car hold 6 people across each row of seats. The roller coaster car can seat 132 people. How many rows of seats does the roller coaster car have? A 20 B 21 C 22 D 23

5. 126 ÷ 3 = A 40 B 42 C 42 R 1 D 43	6. What is the first digit in the quotient of 735 ÷ 5? A 1 B 4 C 5 D 7
7. Ms. Ling is organizing 192 science magazine articles she has saved over the years. She has 8 folders. How many articles should she put in each folder so that each folder holds the same number of articles? A 8 B 18 C 22 D 24	8. There are 224 students in a school marching band. The students march in 8 rows of equal size. How many students are in each row? A 24 B 28 C 36 D 42
9. Yolanda read a 304-page book in 8 days. She read the same number of pages each day. How many pages did she read each day/ A 28 B 38 C 48 D 2432	10. A variety show at the fairgrounds has 5 equal rows of seats. If 150 people can watch the show at one time, how many seats are in each row? A 25 B 30 C 50 D 75

Fourth Grade End of Unit Division Test, page 3 Name _____

11. Mr. Simpson divided his class of 25 students into groups of equal size. How many groups did he create? A 2 B 4 C 5 D 12	12. Colleen has 34 T-shirts. She puts the same number of T-shirts in each of her bags. If she does not have any T-shirts left over, how many bags does she have? A 2 B 3 C 4 D 5
13. What type of number is 9? A prime B composite C mixed D even	14. Which number is evenly divisible by 5? A 32 B 54 C 71 D 80
15. Which of these is another way to write the product of 12 X 6? A 12 X 4 X 2 B 4 x 8 x 6 C 3 X 2 X 6 D 3 X 4 X 6	16. What are all the factors of 36? A 1, 36 B 1, 3, 4, 9, 12, 36 C 1, 6, 36 D 1, 2, 3, 4, 6, 9, 12, 18, 36

17. Kevin wants to list the factors of 12. His list includes 1. 2, 6, and 12. What factors is he missing? A 3, 9 B 8, 4 C 3, 4 D 5, 7	18. Which statement is true? A The only factors of 12 are 1 and 12. B The only factors of 13 are 1 and 13. C The only factors of 14 are 1 and 14. D The only factors of 15 are 1 and 15.
19. Which is a prime number? A 2 B 4 C 6 D 8	20. Which is a prime number? A 14 B 21 C 33 D 47

When you finish the test, go back and check your work, then begin working on the Performance-based Assessment.

Teacher's Answer Key

Fourth Grade End of Unit Division Test

1. B 2
2. B 400
3. A 70
4. C 22
5. B 42
6. A 1
7. D 24
8. B 28
9. B 38
10. B 30
11. C 5
12. A 2
13. B composite
14. D 80
15. D 3 X 4 X 6
16. D 1, 2, 3, 4, 6, 9, 12, 18, 36
17. C 3, 4
18. B The only factors of 13 are 1 and 13.
19. A 2
20. D 47

4th Grade--Division

Final Performance Task

Name: _____

Think of a situation describing the following problem:

$287 \div 14 =$

Write a story problem and then solve it. As you solve the problem, record each step of your work so someone looking at your work would understand your thinking

Rubric for Mathematics Performance Task

	4 **Exemplary**	**3** **Adequate**	**2** **Somewhat**	**1** **Minimal**
Creates appropriate problem situation	A creative problem situation	The problem fits the situation	Problem rambles or is hard to follow	The problem is nonsensical
Records all steps	Detailed description of all steps	All steps are listed in sequence	3 steps listed, or out of sequence	1 or 2 steps listed
Deals with remainders appropriately	The precise use of the remainder, with explanation	Remainder used correctly without explanation	Remainder is listed as R ___	Remainder is not mentioned, ignored
Uses efficient strategies	More than one strategy is evident	Uses one strategy completely	False start or incomplete strategies	Counting, drawing relied upon to solve

Name: _____

Menu Activities after Lesson 4

- ☐ **Choice 1: Array Game: Multiplication Pairs**
- ☐ **Choice 2: Array Game: Count and Compare**
- ☐ **Choice 3: Array Game: Small Array/Big Array**
- ☐ **Choice 4: Hungry Ants**

Menu Activities after Lesson 9

Games

- ☐ **Choice 5: The Game of Leftovers**
- ☐ **Choice 6: Mystery Numbers**
- ☐ **Choice 7: Remainder Face Off, p.133**

Chapter 7 Math Centers

- ☐ **Choice 8: Know Your Nines**
- ☐ **Choice 9: Roomy Dimensions**
- ☐ **Choice 10: What's My Fact**

Chapter 13 Math Centers

- ☐ **Choice 11: Remainders Rule**
- ☐ **Choice 12: Bits and Pieces**
- ☐ **Choice 13: Dividend Rolls**

Menu Activities after Lesson 14

Chapter 14 Math Centers

- ☐ **Choice 14: Division Day**
- ☐ **Choice 15: Division Puzzles**
- ☐ **Choice 16: Divide and Score**

Chapter 15 Math Centers

- ☐ **Choice 17: Flowers Factors**
- ☐ **Choice 18: Prime Time**
- ☐ **Choice 19: Making Trees**